MATHEMATICAL MODELLING OF AGGRADATION IN OPEN CHANNEL EXPANSIONS WITH MOVABLE BEDS

by

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CERTIFICATE

It is certified that the work contained in the thesis entitled *"Mathematical Modelling of Aggradation in Open Channel Expansions with Movable Beds"*, by Pranab Kumar Mohapatra, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

This work presents analytical and numerical models for the analysis of flow and bed level variation in gradual open channel expansions with movable beds. The analytical solutions are obtained for the equilibrium bed profile and corresponding flow parameters based on the one dimensional flow assumption. Cartesian coordinate as well as a cylindrical coordinate system (radial flow) are used for this purpose. The equilibrium bed profile corresponds to steady flow and no further aggradation. The numerical model solves the governing partial differential equations for water flow and sediment transport by finite-difference methods. A quasi-steady uncoupled approach is adopted, wherein the flow equations and the equations for sediment transport are solved separately. Flow conditions are assumed steady during the solution of equations for sediment flow. One-dimensional as well as two-dimensional models are developed. The two-dimensional models solve for flow parameters by the false transient method . Two numerical schemes , Lax-diffusive and MacCormack scheme are used . A simple algebraic coordinate transformation technique is used to convert the expanding physical domain into a rectangular computational domain to facilitate easy application of boundary conditions. The numerical models predict both temporal and spatial variation in bed levels. All the models presented here are valid only for uniform size sediment and for flows without separation. A comparison between the numerical results and the analytical results for the equilibrium bed profile shows that the numerical models correctly solve the governing

equations. A sensitivity analysis is also performed to study the numerical effects. A parametric study is conducted to investigate the effect of various parameters on aggradation in open channel expansions.

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LIST OF SYMBOLS

a_1, a_2	Sediment Transport Parameters
A	Cross-sectional Area of the Channel
b	Distance of Channel Wall from Center line at any Distance
b'	Differential Coefficient of b w.r.t. x
B	Channel Width at any Distance
B'	Differential Coefficient of B w.r.t. x
BI	Channel Width at Inlet
BO	Channel Width at Outlet
$C_1 - C_{12}$	Constants
F	Any Variable Used in the Numerical Scheme
g	Acceleration due to Gravity
gs	Total Sediment Discharge per unit Length
G	Total sediment discharge
h	Flow Depth at any Section
h_d	Downstream Flow Depth
n	Manning's Roughness Coefficient
q	Flow Discharge per unit Length
Q	Flow Discharge
r	Radial Distance
r_i	Radius at the start of the Expansion
r_o	Radius at the end of the Expansion
SO	Bed Slope
SF	Friction Slope

t	time
V	Velocity of Flow at any Section
V_r	Velocity of Flow at any Section along r
x	Longitudinal Direction
X	Sediment Transport Parameter
y	Transverse Direction
Y	Flow Parameter
Z	Depth of Sediment w.r.t. a Reference Line
ZI	Initial Bed Level at Upstream End
α	Angle Between Velocity at wall and x axis
β	Angle of Expansion
γ	$1/a_2$
η	Transformed y- direction
λ	Porosity of Bed
θ	Angle between Center Line and Wall = Half of β
ξ	Transformed x- direction
Subscripts x,y	Denote the parameter in that Direction
Subscripts i,j	Refer to numerical Grid points along ξ and η directions respectively
Superscripts k	Denotes the Variable at time level k
Superscripts *,**	Refer to Predictor and Corrector part Values in MacCormack scheme

CHAPTER I

INTRODUCTION

An open channel transition is a channel reach in which cross-section or slope changes with distance such that there is a change in the flow parameters (Henderson 1966). Channel expansions and contractions are common examples of such transitions. They are usually required in human made channels for several practical purposes.They are used in structures between canals and flumes,canals and tunnels and canals and inverted syphons to reduce energy losses(Chow 1959). Many hydraulic structures such as barrages and bridges may require constriction of the channel in order to reduce the length of the structure and thereby the cost. These structures require contractions on the upstream side and expansions on the downstream side. This is especially true in case of sediment-laden flows because channel bed may aggrade or degrade depending on the flow conditions. Understanding of flow phenomenon in these transitions and ability to quantify the variations in flow parameters is essential to the proper design and operation of hydraulic structures such as syphons, aquaducts, barrages, bridges etc. Expansions and contractions may also occur naturally. For example, rivers expand and widen before they enter reservoirs, oceans or other main rivers. They also widen when they leave steep mountainous areas and enter gentle plains. Flow analysis in such natural expansions helps in our understanding of

reservoir sedimentation and delta formation.

In an expansion, the flow area increases in the downstream direction and as a consequence the velocity decreases.Therefore,the sediment carrying capacity gets reduced as the water enters an expansion and this results in sediment deposition.An expansion built with a uniform slope initially, will aggrade till it attains an equilibrium bed level.Such a state is reached when the sediment carrying capacity is same at every cross-section in the expansion.The above simple picture gets complicated for the case of large angle and sudden expansions.The flow separation and the attendant recirculation zones change the flow characteristics and sediment movement significantly.Infact, Nashta et al.(1987) have observed erosion at the entrance region of a sudden channel expansion with movable bed.The process gets even more complicated if the sediment is not uniformly sized.The larger particles tend to move towards the wall(Nashta et al.1987) and the finer particles get deposited downstream.During degradation, finer particles are transported in preference to larger particles. There may be a stage when the bed is predominantly made up of large particles which may provide a protective layer for the underlying finer particles. This process is called armouring of bed.

The analysis of flow and bed variation in expansions is complicated because the flow is two-dimensional. Difficulties are also encountered if the flow is supercritical and shocks are present. The governing partial differential equations do not have analytical solutions for majority of cases and therefore, require numerical solution. Application of boundary conditions in these

numerical schemes is not straight forward and may pose difficulties. The time scale of flow variation is significantly different from the time scale of bed level variation and this should be taken into account while developing an efficient numeriacal algorithm. Even if the numerical scheme is efficient and reliable, uncertainty in the computational results arises because the relationship between water and sediment still defies regorous treatment from the point of view of basic mechanics.

The present work is an attempt to study the bed level variations in open channel expansions with movable beds. It is limited to subcritical flow in gradual expansions where there is no flow separation and to beds comprising of uniform size sediments.

1.1 REVIEW OF LITERATURE

In the past, several attempts have been made to study the flow characteristics in channel expansions.Chaturvedi(1963) and Garde et al.(1979) have studied the subcritical flow in a gradual expansion .Lokrou and Shen (1983) analyzed the flow in sudden expansions by a similarity approach while Mehta(1979) studied the flow characteristics in two-dimensional expansions. Hinds (1928), Mitra(1940) and Chaturvedi (1963) proposed different design procedures for expansions with given flow conditions and certain assumptions based on experiments .Vittal and Chiranjeevi (1983) proposed a rational method of design for open channel transitions.Chow(1959),Scogerboe et al.(1971) and Garde et al.(1979) attempted to design devices to control separation in expanding channels .

Most of the studies on open channel expansions are with respect to fixed-bed channels.Studies on movable bed expansions are only few. Ashida and Miyai(1964) experimentally analysed the formation of deltas in an abrupt expansion .Bates (1953) studied the bar formations in streams as they come out of a natural constriction . Chang(1982) and Mertens(1986) (Raudkivi 1990) described experimental data on aggradation in the form of delta formation in reservoirs. It is found that deltas in reservoirs grow both in length and width. The initial development is a strongly elongated delta in flow directions,over which the flow depth shallows. This encourages sidewards flow. The other effect is rise in the water level in the channel upstream of the reservoir. The reservoir in the above experiments is modelled as a sudden expansion. Figure 1.1 shows the formation of sand delta in

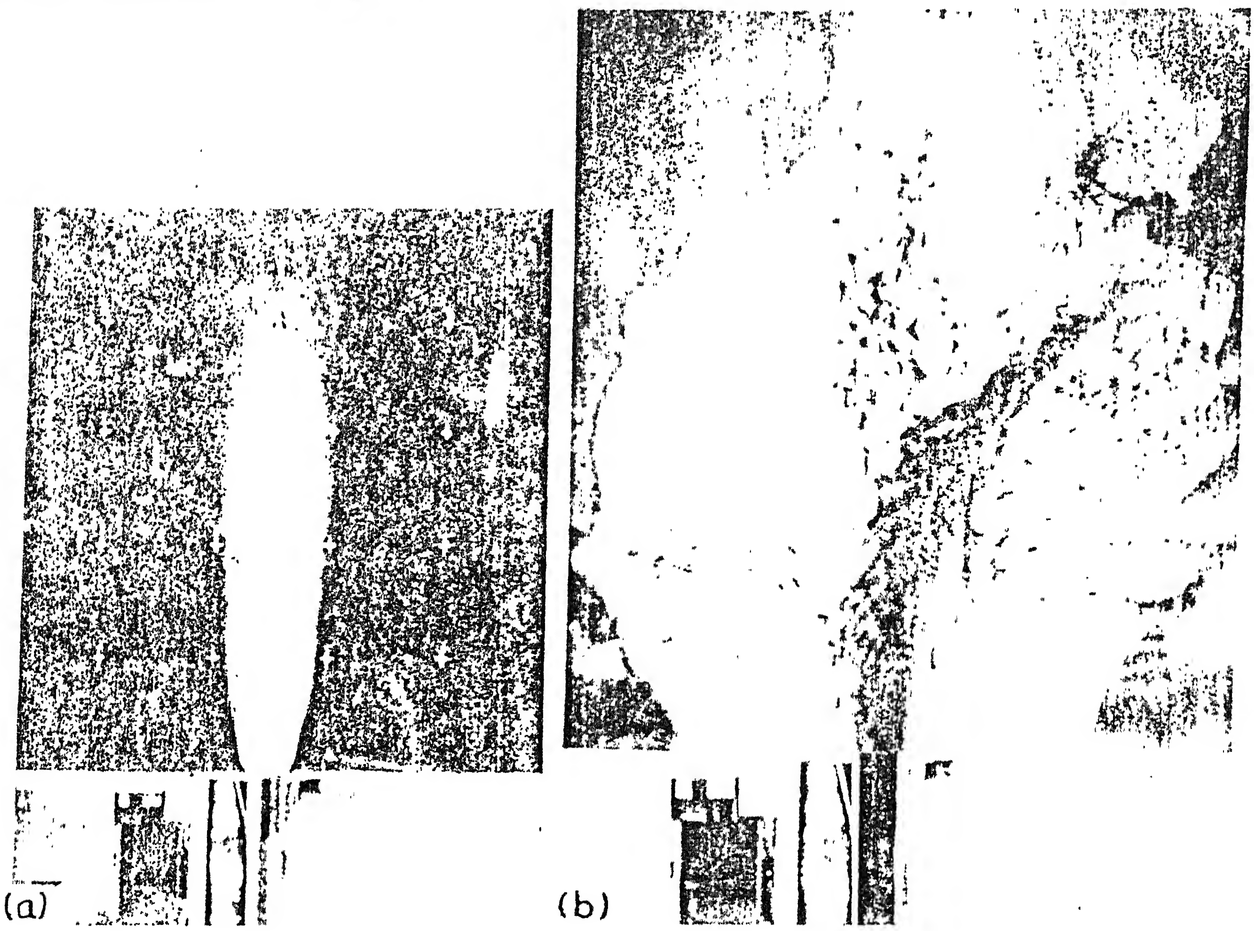
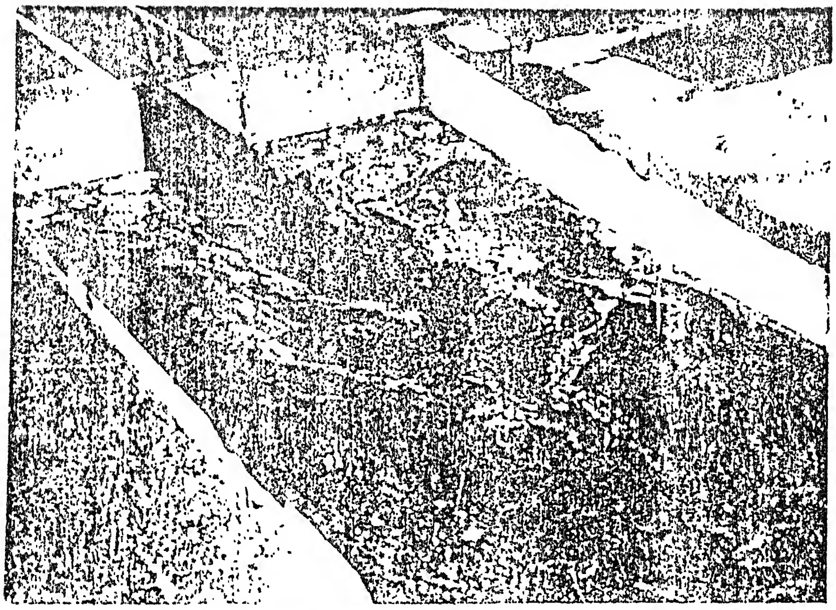


Figure 1.1 Formation of a sand delta in laboratory experiment, (a) after 2 hours, (b) after

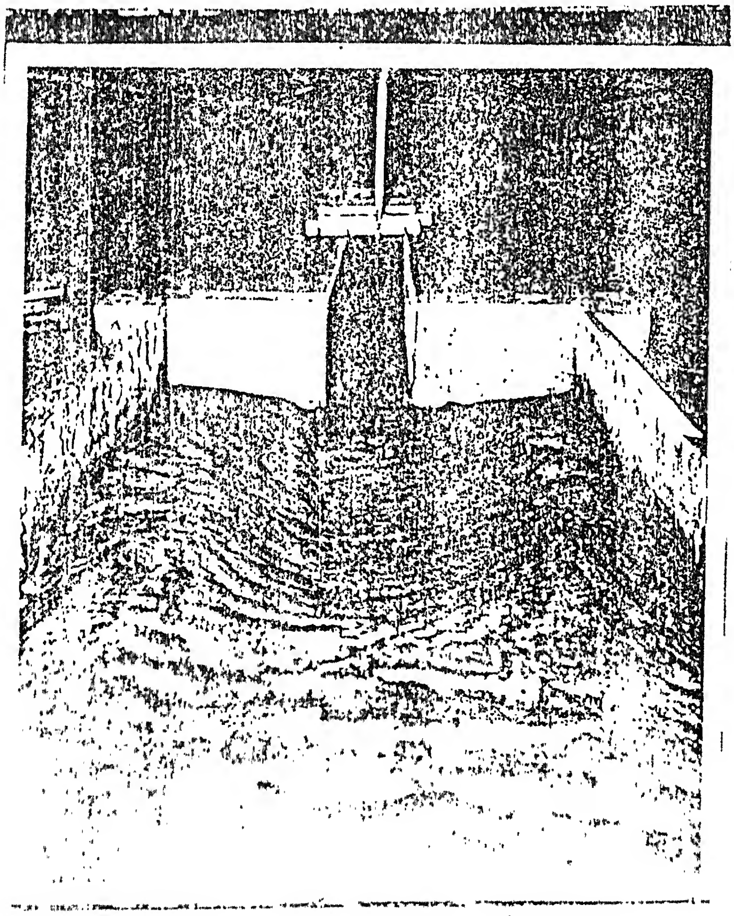
laboratory experiments.Recently,Nashta(1984) and Nashta et al.(1987) investigated subcritical flow in abrupt expansions with movable beds.They found that the high velocity jet scours the material of the bed.The curvature of the separating stream line makes the coarser sediment to move and deposit towards the walls. The finer material is transported downstream and is deposited in the form of a low transeverse bar. Figure 1.2 shows the flow pattern and the bed topography in a sudden expansion with movable bed. The available experimental work in movable bed expansions is mostly related to sudden expansions and studies on gradual expansions are not many.

Laursen (1958,1963) derived analytical solutions for scour depth in long contractions. He solved the flow equations as given by the Manning's formula along with a sediment discharge relationship for the condition of final equilibrium bed profile. His solution is valid only for predicting the maximum scour depth.
It does not describe the general two-dimensional bed level variation.Similar approach may be used for channel expansions. Straub(1953) also used similar approach but with a different sediment transport relation.

Previous studies of open channel expansions have been mostly either experimental or dealt with the analytical solutions to simple cases.Also, no attempts have been made to study the temporal variations of bed level in an expansion due to non-uniform flow conditions.Although experimental and analytical studies contribute significantly to our understanding of open channel flows with movable beds, mathematical modelling (using



FLOW PATTERN IN MOVABLE BED EXPANSION ($B_2/B_1=4.5$, $Q=0.012\ m^3/s$)



BED TOPOGRAPHY IN MOVABLE BED EXPANSION ($B_2/B_1=4.5$, $Q=0.012\ m^3/s$)

Figure 1.2

numerical shemes) of movable bed hydraulics has become an active area of research in computational hydraulics.Mathematical models for movable bed channels essentially solve the governing equations for water and sediment flow. Water flow is usually represented by the shallow water equations which describe the conservation of mass and momentum.The sediment flow is represented by the sediment continuity equation and a sediment transport equation which relates the sediment discharge to the flow parameters.This sediment transport equation is generally an empirical equation based on laboratory and field investigations . The above governing equations for water and sediment flow constitute a set of non-linear hyperbolic partial differential equations for which analytical solutions are possible for only simple cases.

Therefore,they are solved using numerical schemes . Either finite-difference or finite-element methods can be used for this purpose.Most of the standard movable bed models can be classified into either uncoupled models, wherein the water-flow equations and sediment continuity equation are uncoupled during a given time step or coupled models, wherein all the governing equations are solved simultaneously.They can also be classified as either unsteady models or quasi-steady flow models.The unsteady models solve the complete Saint Venant equations describing the water flow.On the other hand, quasi-steady flow models assume water flow to be steady during the computation of bed level variation.Models also differ with regards to the physical conception and the numerical scheme employed for the solution.Some models use uniform size sediment while others use non-uniform size sediment and attempt to

simulate armouring process also.For the modes of transport of sediment,some use total load concept while others make a distinction between different mode of transport. A detailed classification of models is presented in figure 1.3 .

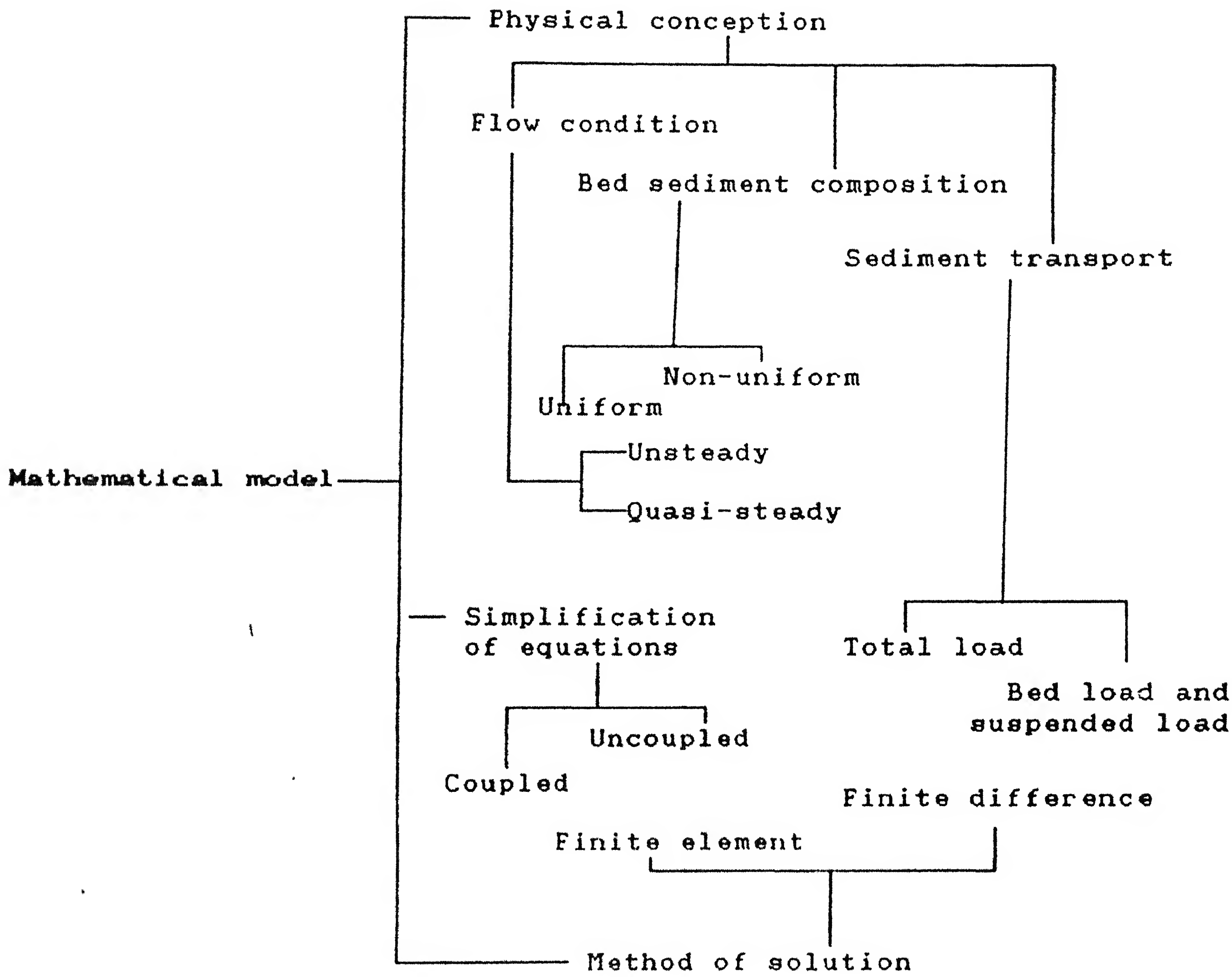


Figure 1.3 Model classification

Although the achievements in the fixed bed modelling (bed level is assumed invariant) are considerable,same can not be expected of movable bed models.The fixed-bed open channel flow is well described by partial differential equations

developed by Saint Venant in 1871.The numerical solution of these equations and the results obtained therefrom have consolidated the base of computations in fixed-bed flows.On the contrary, the physics of the movable bed flows is poorly understood.The understanding is not satisfactory even for conditions like one-dimensional flow with uniform size sediments. In practical situations,the sediment particle size is non-uniform and the flow is at least two-dimensional if not three-dimensional.Besides, the armouring and sorting effects may be present.However,considering the time scale,the cost and difficulty in measuring the flow parameters and bed levels in field investigations,mathematical modelling is sometimes preferred to experimental investigation.Also, mathematical models are more convenient to use and several alternatives can be studied easily.But, the model has to be reliable,robust,accurate and economical (Holly 1986).

Since the early work of Vreugdenhil and de Vries in 1967 (Cunge et al.1980),several one-dimensional movable bed models have been developed.Holly(1986) and Dawdy and Vanoni(1986) presented excellent reviews of numerical simulation of alluvial hydraulics.HEC-6,developed by U.S. Army Corps of Engineers,FLUVVIAL-3(Chang and Hill 1976),IALLUVIAL(Karim and Kennedy 1982) and CHAR-2 (SOGREAH 1978) are some of the commercially available movable bed models for one-dimensional flows.HEC-6 is a good example of a quasi-steady flow model whereas CHAR-1 (Cunge et al. 1980) is an unsteady model.

One-dimensional movable bed models are many.However,only few two-dimensional movable bed models are available.The models developed by de Vriend and Struiksma(1983)

and Shimizu and Itakura(1989) are applicable only for curved channels.Matsutomi et al.(1984) developed an unsteady model for routing dam break floods in natural channels with movable beds.However,it has not been applied to study the long term bed level changes in gradually expanded channels.TABS-2 (Waterways Experiment Station 1987) is a finite element model for water and sediment routing .It solves unsteady shallow water equations and sediment continuity, and is primarily designed for analysing sedimentation in large bodies of water like estuaries,reservoirs etc.It is not efficient for analysing long term bed evolution in channel transitions which are usually not very long.

1.2 SCOPE OF THE PRESENT STUDY

The literature review presented in the previous section shows that majority of experimental studies on channel transitions with movable beds are for sudden expansions.There are only few studies on gradual expansions.Although a number of mathematical models are available for movable bed hydraulics,most of them are for one-dimensional flows.The available two-dimensional models are either not applicable to the case of movable bed expansions or inefficient.

The scope of the present work is to

1) develop a two-dimensional mathematical model for analysing long term bed evolution in gradual expansions
2) develop an appropriate one-dimensional movable-bed model which can be applied to gradual expansions
3) develop analytical solutions to the equilibrium bed profile in

an expansion by assuming the flow to be radial so that the mathematical models can be validated and

4) to study the aggradation characteristics in a gradual expansion through a parametric study.

The numerical models presented in this study use a quasi-steady uncoupled approach for simulating long term bed evolution in gradual expansions.The false transient method is employed to obtain the steady flow parameters.Two numerical schemes, Lax-diffusive scheme and MacCormack scheme are used for this purpose.The application of the models is limited to subcritical flows in gradual expansions where there is no flow separation. Also, the bed is assumed to be made of uniform size particles.

Chapter II presents the governing equations and the assumptions made in deriving them. Analytical solutions are derived for few simple cases in chapter III . The numerical schemes for the solution of the governing equations are described in chapter IV. Chapter V discusses the computational results and the conclusions are presented in chapter VI .

CHAPTER II

GOVERNING EQUATIONS

To simulate the unsteady flow in open channels with movable bed, the governing partial differential equations for the flow of water and sediment are numerically solved. Although extensive research has been carried out to understand the exact relationship between water flow and sediment movement (Shen 1970,Garde and Rangaraju 1985),the present knowledge in this area can only be considered semi-empirical. Also, complete solution of three-dimensional equations of motion for water and sediment is very complicated. Since the aim of the present study is to simulate only the bed level variation in a channel expansion, the following two major assumptions are made in the analysis.

(1) We are not interested in computing the velocity of sediment particles and (2)the sediment discharge at any point is uniquely related to the depth averaged flow parameters at that point.

The above assumptions make the sediment continuity equation sufficient to describe the sediment flow .The equation for the conservation of momentum for the sediment is implicitly represented by the relationship between the sediment discharge and

depth averaged flow parameters.

In this chapter, governing equations for water and sediment are presented. Co-ordinate transformation for transforming the physical domain of an expanded channel into a rectangular computational domain is discussed. Simplified equations for one-dimensional assumption are also discussed.

2.1 WATER FLOW

The two-dimensional unsteady gradually varied flow equations in open channels are (Lai 1977, Jimenez 1987) :

continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial (Vx\ h)}{\partial x} + \frac{\partial (Vy\ h)}{\partial y} = 0 \qquad \text{.........................(2.1)}$$

momentum equation in x-direction

$$\frac{\partial (Vx\ h)}{\partial t} + \frac{\partial (Vx^2\ h + gh^2/ 2)}{\partial x} + \frac{\partial (Vx\ Vy\ h)}{\partial y} = gh(SO_x - SF_x) \qquad \text{.........................(2.2)}$$

momentum equation in y-direction

$$\frac{\partial (Vy\ h)}{\partial t} + \frac{\partial (Vx\ Vy\ h)}{\partial x} + \frac{\partial (Vy^2\ h + gh^2/ 2)}{\partial y} = gh\ (SO_y - SF_y) \qquad \text{.........................(2.3)}$$

in which, h is the flow depth, Vx and Vy are the depth averaged velocities in x-direction and y-direction respectively, g is the

acceleration due to gravity, and SO_x and SO_y are bed slopes in x and y- directions respectively.x and y are co-ordinate axes, and t is time.

The friction slopes are calculated using the following equations .

$$SF_x = \frac{n^2 \; V_x \sqrt{V_x^2 + V_y^2}}{h^{1.333}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.4)$$

$$SF_y = \frac{n^2 \; V_y \sqrt{V_x^2 + V_y^2}}{h^{1.333}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.5)$$

in which, n is Manning's roughness coefficient.

Although n is in general a complicated function of flow depth, bottom roughness,slope, discharge and bed forms, a constant value is assumed throughout this study. However elaborate equations can be easily incorporated because explicit numerical methods are used for the solution of governing equations.

Equation 2.1 - 2.3 are obtained by depth averaging the three-dimensional equations and making the following assumptions

(1)acceleration in the vertical direction is ~~are~~ negligible. In otherwords pressure distribution along the vertical is hydrostatic.

(2)the velocity distribution is uniform over the flow depth.

(3)bottom shear stress is dominant and all other shear

stresses are negligible.

(4)the friction losses computed using steady state formulae are valid even for unsteady flow conditions.

(5)the channel bottom slope is small.

The above assumptions are valid for most of the gradually varied flow situations.However, the governing equations do not account for the effective stresses which arise due to

(i)laminar viscous stresses,

(ii)turbulent stresses and

(iii)stresses due to depth averaging

Extra turbulent,stress-like terms appear while depth averaging the momentum equations because of the non-uniformity of the velocity in the vertical direction .Based on experiments in the laboratory channels, Odgaard and Bergs(1988) have shown that the error introduced by uniform velocity assumption is negligble. Flokstra(1977) also showed that away from the walls the effective stresses are dominated by the bottom stress.However, it should be noted that these effective stresses should be considered while simulating circulating flows (Flokstra 1977). Therefore, models presented in this study are not valid for large angle expansion where flow separation occurs.Few models are available for simulating effective stresses using turbulence closure models.However, these methods,as applied in open channels,(Rastogi and Rodi 1978) are at best successfull for fixed bed channels.Also, their application may be required only if one is interested in three-dimensional flow structure and actual sediment

movement.

2.2 SEDIMENT FLOW

As discussed earlier, only sediment continuity equation and a sediment discharge-water flow relationship are required for completely representing the sediment flow. In cartesian co-ordinate system the sediment continuity equation is given by

$$\frac{\partial Z}{\partial t} + \frac{1}{1-\lambda}\left[\frac{\partial}{\partial x}(gs_x) + \frac{\partial (gs_y)}{\partial y}\right] = 0 \quad \text{.........................(2.6)}$$

Where, Z is the bed elevation, gs_x is sediment discharge per unit length in x-direction , gs_y is sediment discharge per unit length in y-direction and λ is the porosity of the ~~sediment particles~~ bed. gs_x and gs_y depend on the flow parameters at that point. Several relations are available for estimating these. These relations are well documented elsewhere (Graf 1972, Garde and Rangaraju, 1985) and are not described here. As suggested by Vanoni(1975) and Jansen et al.(1979) many of these equations can be represented in the following functional form.

$$X = f\ (\ Y\ ) \quad \text{.................................(2.7)}$$

Where ,X is a transport parameter and Y is a flow parameter. The transport parameter depends on the sediment discharge and the grain properties, while the flow parameters depend on the flow properties and the grain diameter. The numerical

models in the present study are structured in such a way that any sediment discharge relationship can be easily considered. However, the following equation is adopted for its simplicity.

$$gs = a_1 \left[V_x^2 + V_y^2 \right]^{a_2/2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.8)$$

Where, gs is the resultant sediment discharge per unit length, and a_1, a_2 are empirical constants which depend on sediment grain properties, bed forms and flow properties. Although the above relationship is a highly simplified one , it has been used in mathematical models by many researchers to gain insight into the problem of aggradation and degradation (Gill 1983, Zhang and Kahawita1987, Park and Jain 1987). Analytical solution for bed level variations in a channel expansion could be obtained by the use of above equation. This made it possible to verify the numerical models by comparing the numerical results with the analytical solution. Before closing this discussion, it should be mentioned that the above equation is valid only for uniform sediment size. Therefore, the models do not give proper results in case of degradation where armouring effect is present. During degradation, finer particles are transported in preference to larger particles. There may be a stage when the bed has only sufficiently large particles which can provide a protective layer for the underlying finer particles. This protective layer is refered to as the armoured layer. It should also be mentioned at this stage that no distinction is made between the bed load and the suspended load and equation 2.8 is assumed to represent the

total sediment discharge. The aim of the study is to develope a numerical model for analysing aggradation in channel expansions and validate the numerical model by comparing the numerical results with the anlytical results. Therefore, the use of a simplified relationship is justified. However, a more suitable sediment discharge relationship should be included and the model should be calibrated before it is used in field situations. As mentioned earlier, any sediment discharge relationship can be easily incorporated into the models presented in this study.

2.3 CO-ORDINATE TRANSFORMATION

Equations 2.1, 2.3 and 2.6 are for a cartesian co-ordinate system. Therefore, it becomes necessary to replace the boundaries of an expansion by an approximate grid (figure 2.1) to solve the governing equations by finite difference techniques.

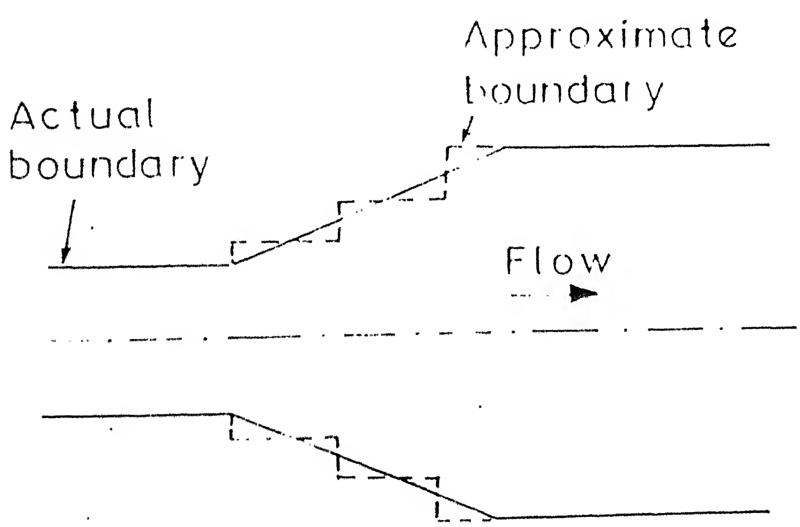
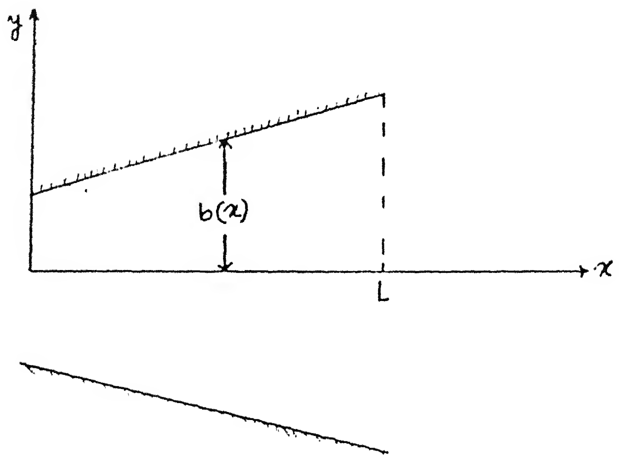
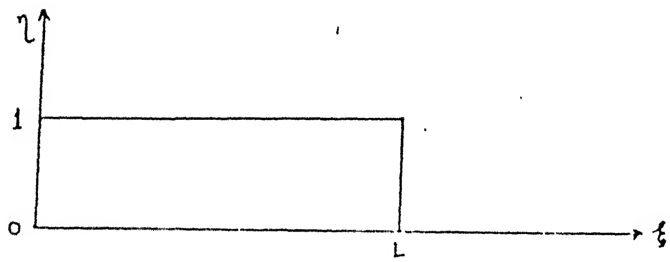


FIGURE 2·1 BOUNDARY APPROXIMATION



(a) physical domain



(b) computational domain

Figure 2.2 Coordinate transformation

This approximation may adversely affect the solution (Roache 1972). This may be taken care of by transforming the physical domain to a rectangular computational domain (figure 2.2) using the following algebraic transformations (Anderson et al. 1984)

$$\xi = x \quad \text{.....................................(2.9)}$$

$$\eta = \frac{y}{b(x)} \quad \text{.....................................(2.10)}$$

Where, b(x) is the distance between the symmetryline and the upper boundary (figure 2.2-a).Only half of the transition is to be computed because of symmetry . Now the boundaries of the computational domain coincide with $\eta = 0$ and $\eta = 1$ (figure 2.2-b).The governing equations need to be transformed so that they can be used with reference to the rectangular computational plane.Applying the chain rule of differentiation

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta} \cdot \frac{\partial \eta}{\partial x}$$

$$= \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta} \left[y \; (-1/b^2) \; b'(\xi) \right]$$

or $$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} - \frac{\eta \; b'(\xi)}{b(\xi)} \cdot \frac{\partial}{\partial \eta} \qquad \text{.........................(2.11)}$$

where, $b'(\xi) = \dfrac{\partial b(\xi)}{\partial x}$

Similarly,

$$\frac{\partial}{\partial y} = \frac{1}{b(\xi)} \frac{\partial}{\partial \eta} \qquad \text{..............................(2.12)}$$

Substitution of equation 2.11 and 2.12 in equations 2.1, 2.2 ,2.3 and 2.6 and subsequent simplifications lead to

continuity equation for water

$$\frac{\partial}{\partial t}[\ b(\xi)\ h] + \frac{\partial}{\partial \xi}[b(\xi)\ V_x\ h\ ] + \frac{\partial}{\partial \eta}\left[V_y\ h - \eta\ b'(\xi)\ V_x\ h\ \right] = 0$$

..........................(2.13)

momentum equation for water in x-direction

$$\frac{\partial}{\partial t}[b(\xi)\ V_x\ h\ ] + \frac{\partial}{\partial \xi}\left[\ b(\xi)\ (h\ V_x^{2} + gh^2/2)\right] +$$

$$\frac{\partial}{\partial \eta}\left[V_x\ V_y\ h - \eta . b'(\xi)\ (h\ V_x^{2} + gh^2/2)\right]$$

$$= \cancel{b(\xi)}\ g\ h \cdot \left[-\frac{\partial}{\partial \xi}\{Z\ b(\xi)\} + \frac{\partial}{\partial \eta}\{\eta\ b'(\xi)\ Z\} - SF_x b\right]$$

........................(2.14)

momentum equation for water in y-direction

$$\frac{\partial}{\partial t}[b(\xi)\ V_y\ h\ ] + \frac{\partial}{\partial \xi}\left[\ b(\xi)\ V_x\ V_y\ h\right]$$

$$+ \frac{\partial}{\partial \eta}\left[(h\ V_y^{2} + g.h^2/2) - V_x\ V_y\ h\ \eta\ b'(\xi)\ \right] = b(\xi) g\ h\left[\frac{-1}{b(\xi)}\frac{\partial z}{\partial \eta} - SF_y\right]$$

........................(2.15)

continuity equation for sediment

$$(\ 1-\lambda\ )b(\xi)\ \frac{\partial Z}{\partial t} + \frac{\partial}{\partial \xi}\left[\ b(\xi) . gs_x\right] + \frac{\partial}{\partial \eta}\left[gs_y - \eta\ b'(\xi) . gs_x\right] = 0$$

.......................(2.16)

Note that the bed slopes SO_x and SO_y are replaced by the following

$$SO_x = -\frac{\partial Z}{\partial x} \quad \text{and,} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.17)$$

$$SO_y = -\frac{\partial Z}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.18)$$

It should also be noted that the physical domain is not a function of time i.e. the water surface width is not changing with time. Therefore, the application of the model is limited to channel cross-sections with vertical side walls.

2.4 ONE-DIMENSIONAL GOVERNING EQUATIONS

The one-dimensional governing equations can be obtained from the two-dimensional equations by averaging them for any cross-section and assuming that the velocity, the bed level and flow depth do not vary across the width. These are given as below:

continuity equation for water

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.19)$$

momentum equation for water

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(Q^2/A) + g\, A \frac{\partial h}{\partial x} = g\, A (SO - SF) \quad \ldots\ldots\ldots\ldots (2.20)$$

sediment continuity equation

$$\frac{\partial}{\partial t}\left[Z\ B\ (1-\lambda)\right] + \frac{\partial}{\partial x}(B\ G) = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.21)$$

The sediment transport and resistance equation will be

$$G = a_1\ (Q/A)^{a_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.22)$$

$$SF = \frac{Q^2 n^2}{B^2\ h^{3.333}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.23)$$

In the above equations , Q = flow discharge, G = sediment discharge and B = channel width. These are well suited for rectangular channels with high values of inlet to outlet width ratio. However, the assumption is not valid for lower values of inlet to outlet width ratio. In this case a better approximation would be to assume that the flow is radial. Refering to figure 2.3

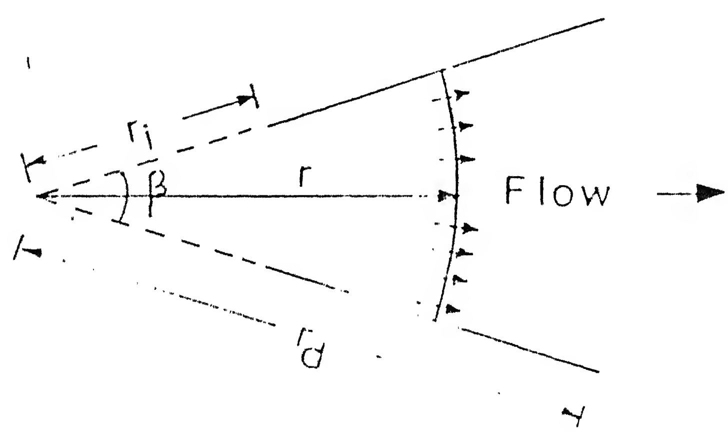


FIGURE 2.3 DEFINITION SKETCH OF RADIAL FLOW

the one-dimensional radial flow equations are (Abbot 1979, Townson and Salihi 1989)

Continuity equation for water

$$\frac{\partial h}{\partial t} + h\,\frac{\partial Vr}{\partial r} + Vr\,\frac{\partial h}{\partial r} + \frac{Vr\ h}{r} = 0 \qquad \text{.........................(2.24)}$$

momentum equation in r-direction

$$\frac{\partial Vr}{\partial t} + Vr\,\frac{\partial Vr}{\partial r} + g\,\frac{\partial h}{\partial r} + g\,\frac{\partial Z}{\partial r} + g.SF = 0 \qquad \text{...............(2.25)}$$

continuity equation for sediment

$$\frac{\partial Z}{\partial t} + \frac{1}{1-\lambda}\left[\frac{\partial G}{\partial r} + \frac{G}{r}\right] = 0 \qquad \text{..............................(2.26)}$$

sediment discharge equation

$$G = a_1\ (\ Vr\ )^{a_2} \qquad \text{...............................(2.27)}$$

the resistance relation will be

$$SF = \frac{Q^2 n^2}{r^2 \beta^2 h^{3.333}} \qquad \text{...............................(2.28)}$$

where, Vr is the velocity in radial direction, r is the radial coordinate and β is the angle of expansion. Though the one-dimensional equations are strictly valid for uniform flow condition across the width or across the arc at a radial

distance, as the case may be, they still provide satisfactory results for flow and bed variation in channel transitions. Also, the equilibrium or steady bed level variation in the channel transition can be obtained by integrating the steady one-dimensional equations. These analytical solutions of the steady one-dimensional equations are presented in the next chapter.

CHAPTER III

ANALYTICAL SOLUTION

The one-dimensional flow and sediment equations are amenable to analytical solution for simple boundary and steady state conditions.A channel expansion with an initially uniform bed slope as shown in figure 3.1 is considered.Even if the downstream and upstream flow conditions i.e. the downstream flow depth and the flow discharge as well as the sediment inflow into the channel are not changed , the channel bed will evolve with time due to the presence of non-uniform flow conditions. The velocity and flow areas are varying in the downstream direction. As a consequence , the sediment inflow into any control volume is not same as the sediment outflow. This imbalance results in either aggradation or degraddation of the channel bed as the case may be . These bed level changes continue till the channel bed attains an equilibrium state such that the sediment carrying capacities of all the sections along the channel are equal. The steady one-dimensional flow and sediment equations can easily be integrated to obtain the above final equilibrium bed profile in an expansion. The derivations based on radial flow as well as rectangular flow assumptions are presented in the following sections.

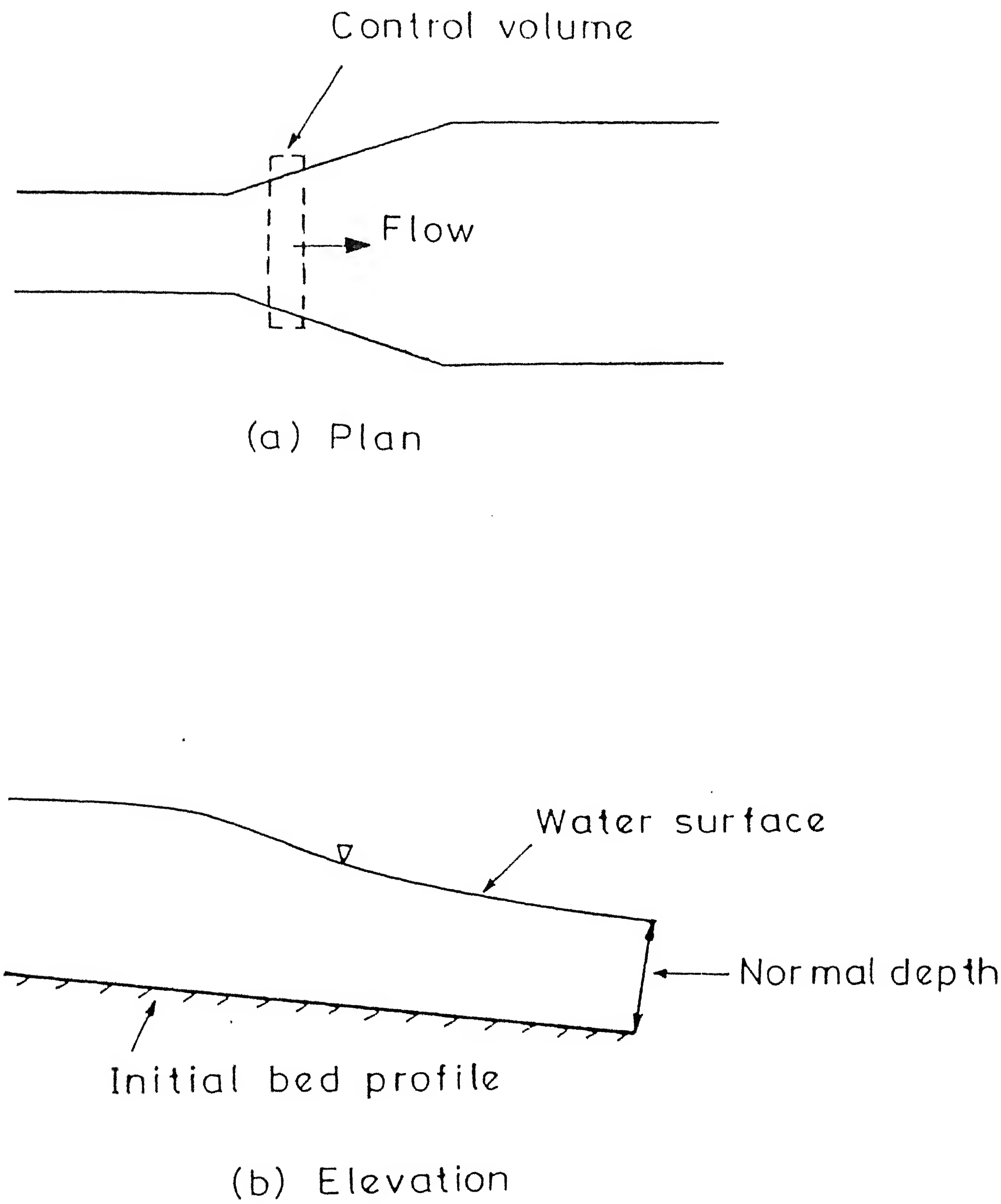


FIGURE 3·1 NON UNIFORM FLOW IN AN EXPANSION

3.1 RADIAL FLOW IN AN EXPANSION

For steady conditions equations 2.24-2.26 become

$$h \frac{dV_r}{dr} + V_r \frac{dh}{dr} + \frac{V_r h}{r} = 0 \quad \ldots\ldots (3.1)$$

$$V_r \frac{dV_r}{dr} + g \frac{dh}{dr} + g \frac{dZ}{dr} + g\, SF = 0 \quad \ldots\ldots (3.2)$$

$$\frac{dG}{dr} + \frac{G}{r} = 0 \quad \ldots\ldots (3.3)$$

Equation 3.1 can be written as

$$\frac{d}{dr}\left[V_r\ h\right] = -\frac{V_r\ h}{r}$$

or

$$\frac{d\left[V_r\ h\right]}{V_r\ h} = -\frac{dr}{r} \quad \ldots\ldots (3.4)$$

Integration of equation 3.4 leads to

$$\log_e (V_r\ h) + \log_e r = \log_e C_1$$

or, $$V_r\ h\ r = C_1 \quad \ldots\ldots (3.5)$$

where, C_1 is the constant of integration.

since,

$$Q = Vr\ h\ (\beta\ r)\ ;$$

$$Vr\ h\ r = \frac{Q}{\beta} \qquad \text{.........................(3.6)}$$

similarly, equation 3.3 gives

$$G\ r = C_2 \qquad \text{.........................(3.7)}$$

where, C_2 is an integration constant.

Substitution of equation 2.27 in equation 3.7 leads to

$$a_1 Vr^{a_2}\ r = C_2 \qquad \text{.........................(3.8)}$$

or,

$$a_1 \left[\frac{Q}{h\ r\ \beta}\right]^{a_2} r = C_2 \qquad \text{.........................(3.9)}$$

or,

$$h = C_3\ r^{1/a_2 - 1} \qquad \text{.........................(3.10)}$$

where,

$$C_3 = \left[\frac{a_1}{C_2}\right]^{1/a_2} \cdot \frac{Q}{\beta} \qquad \text{.........................(3.11)}$$

at the downstream end, $r = r_d$ and $h = h_d$ where, h_d is flow depth at the downstream end.

Substituting the above boundary condition in equation 3.10, we get

$$C_3 = \frac{h_d}{r_d^{1/a_2 - 1}} \qquad \text{.........................(3.12)}$$

Therefore,

$$h = C_4 \, r^{1/a_2 - 1} \quad \ldots\ldots (3.13)$$

where,

$$C_4 = h_d \, r_d^{1-1/a_2} \quad \ldots\ldots (3.14)$$

Substitution of equation 3.13 in 3.6 gives

$$Vr = C_5 \, r^{-1/a_2} \quad \ldots\ldots (3.15)$$

where,

$$C_5 = \frac{Q}{h_d \, \beta \, r_d^{1-1/a_2}} \quad \ldots\ldots (3.16)$$

equation 3.2 may be written as

$$\frac{dZ}{dr} = -SF - \frac{dh}{dr} - \frac{Vr}{g}\frac{dVr}{dr} \quad \ldots\ldots (3.17)$$

from equation 2.28

$$-SF = -\frac{Q^2 n^2}{r^2 \beta^2 h^{3.333}} = C_\sigma r^{1.333 - \frac{3.333}{a_2}} \quad \ldots\ldots (3.18)$$

where, $C_\sigma = -\dfrac{Q^2 n^2}{\beta^2 C_4^{3.333}}$ $\quad \ldots\ldots (3.19)$

equation 3.10 gives

$$-\frac{dh}{dr} = C_7 \, r^{1/a_2 - 2} \quad \ldots\ldots (3.20)$$

where $C_7 = -C_4(1/a_2 - 1)$ $\quad \ldots\ldots (3.21)$

from equation 3.15

$$-\frac{Vr}{g}\frac{dVr}{dr} = C_8 \, r^{-2/a_2 - 1} \quad \ldots\ldots (3.22)$$

2

where $C_8 = \dfrac{C_5^2}{a_2 g}$ ....(3.23)

substitution of equations 3.18, 3.20 and 3.22 in equation 3.17 leads to

$$\frac{dZ}{dr} = C_6 r^{1.333-\frac{3.333}{a_2}} + C_7\, r^{1/a_2-2} + C_8\, r^{-2/a_2-1} \quad \text{..........(3.24)}$$

Integrating equation 3.24 with respect to r, we get

$$Z = C_9 r^{2.333-3.333\gamma} + C_{10} r^{\gamma-1} + C_{11} r^{-2\gamma} + C_{12} \quad \text{...............(3.25)}$$

Where,

$$C_9 = \frac{C_6}{2.333-3.333\gamma} \quad \text{..................................(3.26)}$$

$$C_{10} = \frac{C_7}{\gamma-1} \quad \text{..................................(3.27)}$$

$$C_{11} = \frac{-C_8}{2\gamma} \quad \text{..(3.28)}$$

$\gamma = 1/a_2$

and C_{12} is an integration constant.

At $r = r_i$ (r_i is the radius of the inflow section), Z = ZI (ZI is the bed elevation at the inflow section).

Applying the above boundary condition to equation 3.25, we get

$$Z = ZI + C_9 r^{2.333-3.333\gamma} + C_{10} r^{\gamma-1} + C_{11} r^{-2\gamma} - \left[C_9 r_i^{2.333-3.333\gamma} + C_{10} r_i^{\gamma-1} + C_{11} r_i^{-2\gamma} \right] \quad \text{...............(3.30)}$$

Equations 3.13, 3.15 and 3.30 describe the steady state variation of flow depth, velocity and bed elevation in an expanding channel

with movable bed. These are based on one-dimensional radial flow assumption. The equations for rectangular flow assumption are presented next.

3.2 RECTANGULAR FLOW IN AN EXPANSION

For steady state conditions, equations 2.19, 2.20 and 2.21 are simplified to

$$\frac{dQ}{dx} = 0 \qquad \text{.......(3.31)}$$

$$\frac{d(Q^2/A)}{dx} + g A \frac{dh}{dx} = g A (S0 - SF) \qquad \text{.......(3.32)}$$

$$\frac{d}{dx}\left[B\, a_1 (Q/A)^{a_2}\right] = 0 \qquad \text{.......(3.33)}$$

Integration of equation 3.31 gives

$$Q = C_{13} \qquad \text{.......(3.34)}$$

where C_{13} is a constant of integration.

From equation 3.33

$$a_1 (Q)^{a_2} . \frac{d}{dx}\left[\frac{B}{A^{a_2}}\right] = 0 \qquad \text{.......(3.35)}$$

$$\text{or} \quad B(-a_2) A^{-a_2 - 1} \frac{dA}{dx} + A^{-a_2} . B' = 0 \qquad \text{.......(3.36)}$$

$$\text{where,} \quad B' = \frac{dB}{dx} \qquad \text{.......(3.37)}$$

$$\text{or} \quad -Ba_2 (1/A)(1/A^{a_2})\left[\frac{dh}{dx} B + h B'\right] + A^{-a_2} . B' = 0 .$$

$$\text{or} \quad \frac{dh}{dx} = (\gamma - 1) B' \frac{h}{B} \qquad \text{.......(3.38)}$$

Substitution of equation 2.23 in equation 3.32 gives

$$\frac{d(Q^2/A)}{dx} + g A \frac{dh}{dx} = g A\left[-\frac{dZ}{dx} - \frac{Q^2 n^2}{B^2 h^{3.333}}\right] \qquad \text{.......(3.39)}$$

where, $\quad -\dfrac{dZ}{dx} = SO$ ....(3.40)

or, $\dfrac{-Q^2}{gA^3}\left[\dfrac{dh}{dx}B + hB'\right] + \dfrac{dh}{dx} = -\dfrac{dZ}{dx} - \dfrac{Q^2n^2}{B^2h^{3.333}}$ ....(3.41)

or, $\dfrac{dZ}{dx} = \left[\dfrac{BQ^2}{gA^3} - 1\right]\dfrac{dh}{dx} + \dfrac{Q^2hB'}{gA^3} - \dfrac{Q^2n^2}{B^2h^{3.333}}$ ....(3.42)

Substitution of equation 3.38 in equation 3.42 leads to

$$\frac{dZ}{dx} = \frac{(\gamma-1)B'h}{B}\left[\frac{BQ^2}{gA^3} - 1\right] + \frac{Q^2hB'}{gA^3} - \frac{Q^2n^2}{B^2h^{3.333}} \quad \text{...........(3.43)}$$

Integration of equation 3.38 gives

$$h = C_{14}\,B^{\gamma-1} \quad \text{.................................(3.44)}$$

where, C_{14} is an integration constant.

Application of the boundary condition $h = h_d$ at $B = BO$ (BO = width of the downstream end) results in

$$h = h_d\left[\frac{B}{BO}\right]^{\gamma-1} \quad \text{................................(3.45)}$$

Integration of equation 3.43 is not straightforward if not impossible. However, it can easily be integrated numerically by using either Modified Euler's method or Fourth order Rangekutta method. While doing so the boundary condition $Z = ZI$ at $B = BI$ is incorporated. Modified Euler's method and Fourth order Rangekutta method for steady open channel flow conditions are explained in detail by Subramanya (1986) and Chaudhry (1990) and therefore, they are not repeated here.

3.3 CLOSURE

In this chapter analytical solutions for steady state equilibrium bed profiles in channel expansions have been presented. However, analytical solutions for the time variation of bed levels are not possible and numerical methods need to be applied . The numerical schemes for one dimensional as well as two dimensional cases will be discussed in the next chapter.

CHAPTER IV

NUMERICAL SOLUTION

The governing equations for unsteady flow of water and sediment transport as discussed in chapter II constitute a set of non-linear hyperbolic partial differential equations (Lyn 1987) . Analytical solutions for these equations are available only for idealised cases . Therefore, they are solved numerically .In this chapter , numerical schemes are presented for the solution of one-dimensional as well as two-dimensional bed level changes with respect to time .

Bed level changes occur at a much slower speed than the flow changes and unsteady terms in water flow equations may not be important while simulating the bed transients (de Vries 1965) . This is especially true for the cases studied here where change in bed level occurs due to non-uniformity of flow parameters rather than the unsteady variation of velocity and depth . Therefore , a quasi-steady uncoupled model can be used for predicting the long term bed level changes which occur in a channel expansion . The quasi-steady state means the flow parameters for water are constant during a small period of bed elevation change due to sediment flow . This model is more efficient than the unsteady coupled model from the point of view of computational costs . In a quasi-steady , uncoupled model , the equations for water flow and sediment transport are solved separately . The

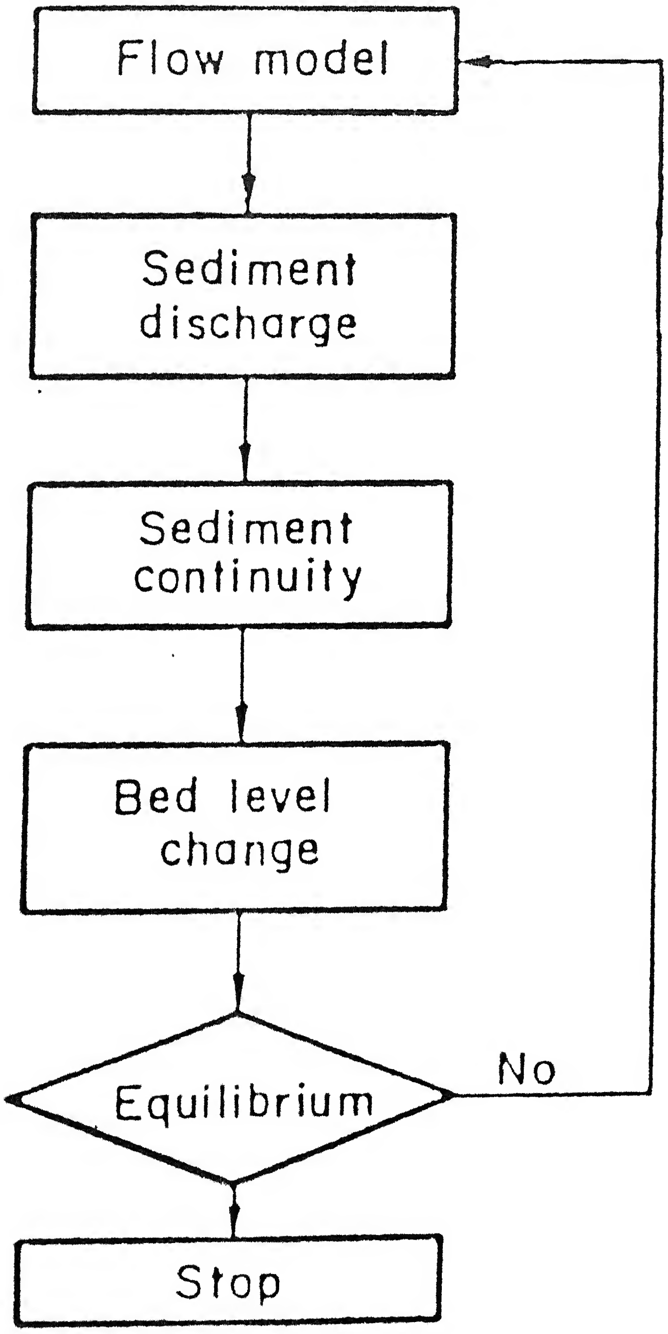
strategy for solution (Fig 4.1) is to first compute the steady flow parameters for computing the sediment discharge values at different sections . These sediment discharge values are then used in sediment continuity equations for computing the bed level changes . After solving the sediment continuity equation , the flow computation is repeated with the new bed configuration and the procedure is continued untill an equilibrium bed profile is reached. .

4.1 COMPUTATIONS FOR ONE-DIMENSIONAL MODEL

For steady state conditions , equation 2.20 may be modified and written as

$$\frac{dh}{dx} = \frac{\dfrac{dZ}{dx} - \dfrac{Q^2 hB'}{gA^3} + \dfrac{Q^2\ n^2}{B^2\ h^{3.333}}}{\dfrac{Q^2 B}{g\ A^3} - 1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4.1)$$

Since Z,Q and B values are known at any x , the right hand side is a function of only h and x . The above equation can be integrated by Modified Euler's method to obtain the h values at any cross section . Subsequently , the velocity in any cross section can also be determined . The numerical integration starts at the downstream end since the h value is known at this section and then proceeds in the upstream direction . The above computed h and V values can be used to determine sediment discharge in each cross section using the equation 2.22 . The sediment continuity



Flow chart for quasi-steady uncoupled model

Figure 4.1

equation (equation 2.21) is represented in the finite difference form to compute the bed level changes as given below .

$$\left.\frac{\partial z}{\partial t}\right]_i = \frac{Z_i^* - Z_i}{\Delta t} \qquad \text{.............................(4.2)}$$

$$\left.\frac{\partial F}{\partial x}\right]_i = \frac{F_i - F_{i-1}}{\Delta x} \qquad \text{.............................(4.3)}$$

where,

$$F=B\ a_1\left[\frac{Q}{B\ h}\right]^{a_2} \qquad \text{.............................(4.4)}$$

In equations 4.2 , 4.3 and 4.4 , the subscript i refers to the node number in the finite difference grid and the superscript * refers to the time level $t+\Delta t$. Substitution of equations 4.2 , 4.3 and 4.4 in equation 2.2 and subsequent simplification results in the following :

$$Z_i^* = Z_i - \frac{\Delta t\ a_1 Q^{a_2}}{\Delta x\ B_i(1-\lambda)}\left[\frac{B_i^{1-a_2}}{h_i^{a_2}} - \frac{B_{i-1}^{1-a_2}}{h_{i-1}^{a_2}}\right] \text{..........(4.5)}$$

Equation 4.5 can be used to determine the bed level at time $t+\Delta t$ at all nodes for $i=2,3..,N+1$ where N is the number of reaches into which the channel is devided . The bed level at node 1 is given by the inflow boundary conditions . In the present study a straight reach is included in the upstream of the expansion

and since there is no change in the sediment inflow , the bed level at node 1 does not vary with time . The new bed levels at time $t+\Delta t$ are substituted in equations 4.1 and the procedure is repeated to obtain the bed levels at time $t+2\Delta t$. The computations are continued till there is no appreciable change in the bed level and the channel attains an equilibrium state .

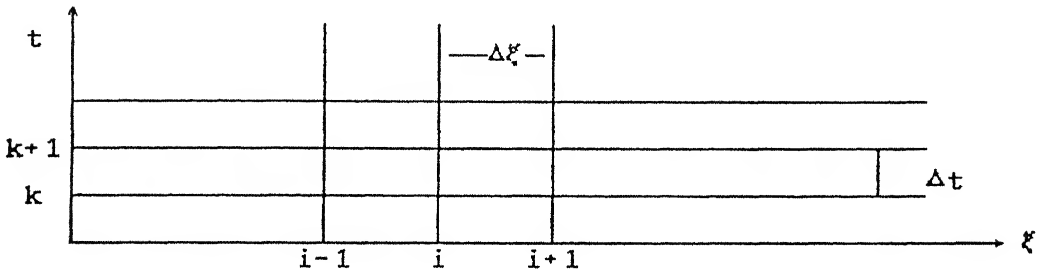
4.2 COMPUTATIONS FOR TWO-DIMENSIONAL MODELS

The two dimensional equations are solved using the same principle as for the one dimensional equations . However , the steady flow parameters in two dimensional model are obtained using the false transient method as described by Bhallamudi and Chaudhry(1991) . In this method , the unsteady water flow equations are solved to obtain the steady state values using time as an iterative parameter . Any initial conditions through the channel are assumed and the boundary conditions are set equal to the actual steady state conditions . Using the two dimensional unsteady flow equations , the system conditions are computed for a sufficient length until the variation of flow variables are negligible and the conditions have converged to the steady state values corresponding to the given boundary conditions . Note that the time in this method is used as an iterating parameter and therefore , the computational time step here is not same as the computational time step Δt used in the sediment continuity equation. In the present analysis , two different finite difference methods are used for water flow computations . These

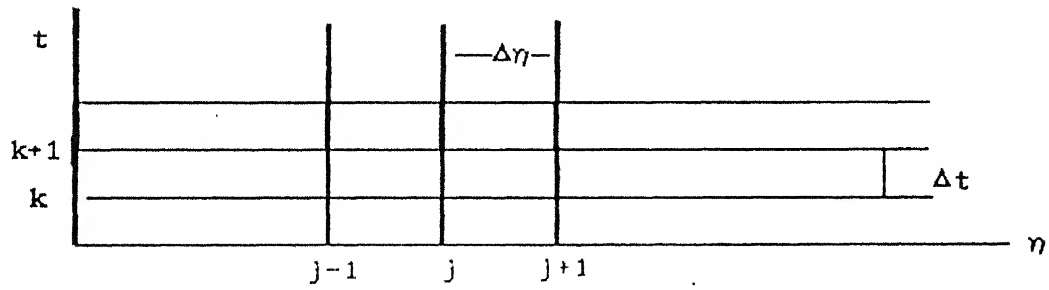
finite difference methods are described in the following sections .

4.2.1 Lax-diffusive scheme

Lax-diffusive scheme is an explicit , first order scheme for solving hyperbolic equations . Refering to figure 4.2 ,



(a) Numerical grid in ξ and t directions



(b) Numerical grid in η and t directions

Figure 4.2 Finite-difference grid

the partial differantial terms in equations2.13 , 2.14 and 2.15 are replaced by the following finite difference approximations :

$$\left.\frac{\partial F}{\partial \xi}\right]_{i,j} = \frac{F^{k}_{i+1,j} - F^{k}_{i-1,j}}{2\ \Delta\xi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.6)$$

$$\left.\frac{\partial F}{\partial \eta}\right]_{i,j} = \frac{F^k_{i,j+1} - F^k_{i,j-1}}{2\,\Delta\eta} \qquad \text{..........................(4.7)}$$

$$\left.\frac{\partial F}{\partial t}\right]_{i,j} = \frac{F^{k+1}_{i,j} - 0.25\left[F^k_{i+1,j} + F^k_{i-1,j} + F^k_{i,j+1} + F^k_{i,j-1}\right]}{\Delta t}$$

$$\text{................(4.8)}$$

$$F\Big]_{i,j} = 0.25\left[F^k_{i+1,j} + F^k_{i-1,j} + F^k_{i,j+1} + F^k_{i,j-1}\right] \qquad \text{.......(4.9)}$$

In equations 4.6 to 4.9 F represents any variable of which the partial differentiation is approximated . In equation 4.9 , F represents the source terms as averaged by the values at neighbouring nodes . The subscripts i,j refer to the grid points in the ξ and η directions respectively . The superscripts k and k+1 refer to the values of the variable at time level k and k+1 respectively . k is the known time level while k+1 is the unknown time level . Substituting equations 4.6 - 4.9 in equtaions 2.13 - 2.15 , algebraic equations are obtained and are solved explicitly for the flow paramaters at the unknown time level .

4.2.1.1 Initial Conditions

To start the unsteady state calculations , values of V_x, V_y, h and Z at time t = 0 are to be specified at all the nodal points . Since a false transient method is used to obtain syteady state conditions , approximate values of V_x , V_y and h are given as initial conditions .

4.2.1.2 Boundary conditions

There are three types of boundaries to be considered in this analysis . These are i) open boundaries , ii) symmetrical boundary and iii) solid side wall boundary .

Open boundaries

An open boundary could be either an inflow or an outflow boundary . The specification of boundary conditions depends on the nature of the flow . Since only sub-critical flows are considered in the present study, two boundary conditions need to be specified at the upstream end and one boundary condition at the downstream end . Flow depth at the down stream end and discharge or V_x at the upstream end constitute these boundary conditions . A straight portion is included upstream of the channel and the flow is assumed to be one-dimensional at the upstream end , Therefore , the transverse velocity , V_y is zero at this point and this forms the second boundary condition . Even after the specification of boundary conditions from the point of view of well posedness of the problem , extra equations need to be used to completely define the values at the upstream and downstream ends . This is due to the finite difference requirements . Strictly speaking , characteristic method (Chaudhry 1987) should be used to determine V_x and V_y at downstream end . However, a simple extrapolation from the values already calculated at the interior nodes using Lax-diffusive Scheme gave satisfactory results .Such an extrapolation procedure has been used successfully by Bhallamudi and Chaudhry (1991) .

Symmetric boundary

Reflection procedure is adopted for the treatment of symmetric boundary (Roache 1972) .Refering to figure 4.3 a row of

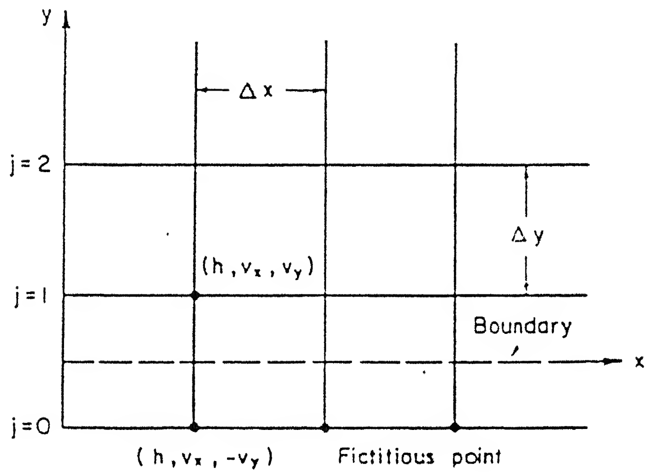


Figure 4.3 Reflection technique for symmetric boundary

imaginary reflection points is considered .All non-conservative flow variables other than the normal velocity are specified as even functions with respect to the symmetric line while the normal velocity is specified as an odd function . This makes the average normal velocity at the boundary equals to zero .

Solid side wall boundary

For the models presented in this study, the slip condition is the proper boundary condition at a side wall since only bottom shear stess is considered . Therefore , the resultant velocity at a solid wall is tangent to it . Anderson et al.(1984), have discussed several boundary techniques in gas dynamics applications. The reflection principle seems to be most suitable

for the present applications. It should be remembered that the reflection technique is only approximate for solid side walls . Refering figure 4.4, the flow depth and the magnitude of the

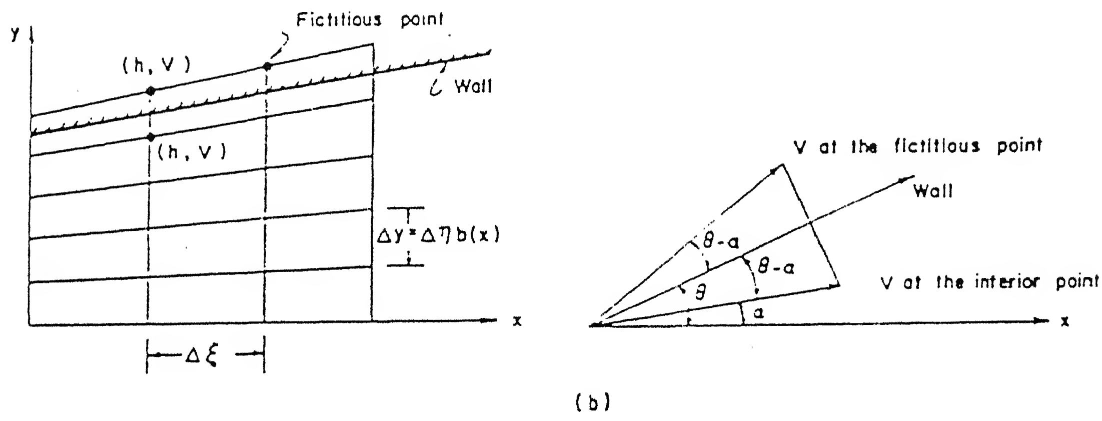


Figure 4.4 Reflection technique for solid side wall boundary

imaginary velocity at reflection point are same as that of corresponding interior grid point . In order to make the normal velocity at wall equal to zero , the velocity components at reflection point are computed using the following formulae .

$$Vx_0 = V\cos(2\theta - \alpha) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.10)$$

$$Vy_0 = V\sin(2\theta - \alpha) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.11)$$

where ,

V= Resultant velocity at the interior point , Vx_0= x-component of the velocity at the reflection point , θ= angle between the wall and x-direction and α = angle between the resultant velocity V and x-direction .

4.2.1.3 Stability

Lax-diffusive Scheme has to satisfy the Courant - Friedrichs - Lewy (CFL) condition for stability . The above conditions in case of transformed equations with transformed co-ordinates may be expressed as :

$$C_{2d} = \frac{\Delta t(V + \sqrt{gh})}{b(x)\ \Delta\xi\ \Delta\eta} \sqrt{\Delta\xi^{2} + \{ b(x)\ \Delta\eta \}^{2}} \leq 1.0 \quad \ldots\ldots.(4.12)$$

equation 4.12 has to be satisfied at every grid point for the scheme to be stable.The above condition is heuristic and is based on a linearised form of the governing equations for one-dimensional flow.It also does not consider the effect of friction and bed slope terms.Therefore, a certain amount of numerical experimentation is required before choosing the upper limit of C_{2d} .

4.2.2 Sediment flow

As already mentioned ,the algorithm uses a quasi-steady flow assumption and therefore, the equation for sediment continuity is solved separately after obtaining the steady flow parameters.The following finite difference approximations are used to solve the sediment continuity equation .

$$\left.\frac{\partial F}{\partial t}\right]_{i,j} = \frac{F_{i,j}^{k+1} - F_{i,j}^{k}}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4.13)$$

$$\left.\frac{\partial F}{\partial \xi}\right]_{i,j} = \frac{F^{k}_{i+1,j} - F^{k}_{i-1,j}}{2\,\Delta\xi} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.14)$$

$$\left.\frac{\partial F}{\partial \eta}\right]_{i,j} = \frac{F^{k}_{i,j} - F^{k}_{i,j-1}}{\Delta\eta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.15)$$

where, F is any variable whose partial derivative needs to be determined. Substitution of equations 4.13 - 4.15 in equation 2.16 gives an explicit equation for `Z' at the unknown time level.

It should be noted again that the Δt used in equation 4.13 is not same as Δt used in the false transient method for water flow. The computational time step used for solving the sediment continuity equation is much larger than the Δt used in the Lax-diffusive Scheme. There is no theoretical work regarding the stability of the quasi-steady uncoupled models for bed-level changes and the computational time step in equation 4.13 is chosen after an extensive numerical experimentation. It should be as large as possible from the point of view of computational efficiency, and at the same time should lead to stable and convergent results.

Before closing this section, it should be mentioned that the upstream boundary condition for the sediment continuity equation is same as that for one-dimensional model . Bed level at the downstream end is obtained by extrapolation from the interior points.

4.2.3 MacCormack scheme

In the present study, the water flow equations are solved by MacCormack Scheme also. This is done to ascertain the accuracy of numerical results. MacCormack Scheme is a second order accurate explicit scheme.It comprises of a predictor part and a corrector part. In the predictor part, forward finite differences are used while in the corrector part, backward finite differences are used for approximating the spatial differential terms.Refering figure 4.2 the predictor and corrector parts are defined as below .

Predictor part :

$$\left.\frac{\partial F}{\partial t}\right]_{i,j} = \frac{F^{*}_{i,j} - F^{k}_{i,j}}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.16)$$

$$\left.\frac{\partial F}{\partial \xi}\right]_{i,j} = \frac{F^{k}_{i+1,j} - F^{k}_{i,j}}{\Delta \xi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.17)$$

$$\left.\frac{\partial F}{\partial \eta}\right]_{i,j} = \frac{F^{k}_{i,j+1} - F^{k}_{i,j}}{\Delta \eta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.18)$$

Corrector part :

$$\left.\frac{\partial F}{\partial t}\right]_{i,j} = \frac{F^{**}_{i,j} - F^{*}_{i,j}}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.19)$$

$$\left.\frac{\partial F}{\partial \xi}\right]_{i,j} = \frac{F^{*}_{i,j} - F^{*}_{i-1,j}}{\Delta \xi} \quad \text{.......................(4.20)}$$

$$\left.\frac{\partial F}{\partial \eta}\right]_{i,j} = \frac{F^{*}_{i,j} - F^{*}_{i,j-1}}{\Delta \eta} \quad \text{.........................(4.21)}$$

Where, F is any variable and i and j refer to nodes in ξ and η directions respectively. Superscripts k is for known time level, * is for predicted values and ** is for corrected values. The value of `F' at the unknown time level k+1 is given by

$$F^{k+1}_{i,j} = 0.5 \left[F^{k}_{i,j} + F^{**}_{i,j}\right] \quad \text{..................(4.22)}$$

Several alternatives for MacCormack scheme are also possible. One can use backward finite difference in corrector part for calculating the spatial variables.Another alternative is to change the direction of differencing from one time step to the next. The initial conditions, boundary conditions and stability criteria are same as discussed for Lax-diffusive Scheme.

In this chapter the numerical schemes were described for the solution of bed level changes and water flow parameters with respect to time. Computer programmes are prepared using the above principles. These programmes are used for analysing sediment deposition in channel expansions. The results of the study are presented in the next chapter.

CHAPTER V

RESULTS AND DISCUSSIONS

In the present study, a two-dimensional, quasi-steady uncoupled model has been used to analyse the variation of flow parameters and bed levels in a gradual expansion with movable bed. As discussed earlier, most of the experimental works in gradual expansions are for fixed beds while most of the studies in movable beds are for sudden expansions. Also, most of the work pertains to the study of equilibrium bed profiles. Experimental data (Soni et al. 1980, Brush and Wolman ~~and Borland~~ 1960, Begin et al. 1981) for temporal variation of bed levels due to unsteady boundary conditions are available only for one-dimensional case. Experimental data are not available for bed level variation in gradual expansions. Therefore, the accuracy of the numerical models developed in this study is checked by comparing the numerical results with the analytical results for one-dimensional case derived in chapter III. Once again it should be borne in mind that proper implementation of boundary conditions and selection of the time steps used in flow equations and sediment continuity equation are important aspects for successful mathematical modelling.

In the present study, five models have been used. For brevity in discussion, these models are refered to as

(i)MODEL 1 : Analytical solution of one-dimensional equations in cylindrical coordinates for equilibrium bed profiles.

(ii) MODEL 2 : Analytical solution of one-dimensional equations in cartesian co-ordinates for equilibrium bed profiles.

(iii) MODEL 3 : Numerical solution of one-dimensional equations in cartesian coordinates for quasi-steady flow and unsteady bed conditions.

(iv) MODEL 4 : Numerical solution of two-dimensional equations for quasi-steady flow and unsteady bed conditions using Lax-diffusive Scheme for solving water flow equations.

(v) MODEL 5 : Numerical solution of two-dimensional equations for quasi-steady flow and unsteady bed conditions using MacCormack Scheme for solving water flow equations.

Models 1,2 and 3 are one-dimensional while models 4 and 5 are two-dimensional. Models 1 and 2 give analytical solution and models 3,4 and 5 give numerical solutions.Therefore, models 2 and 3 form the link between analytical and numerical models.

In the following sections,model verification, sensitivity of the numerical models to numerical parameters and a parametric study for final equilibrium bed profile are presented.

5.1 MODEL VERIFICATION

Model verification has been done in two phases. First, all the one-dimensional models are compared among themselves

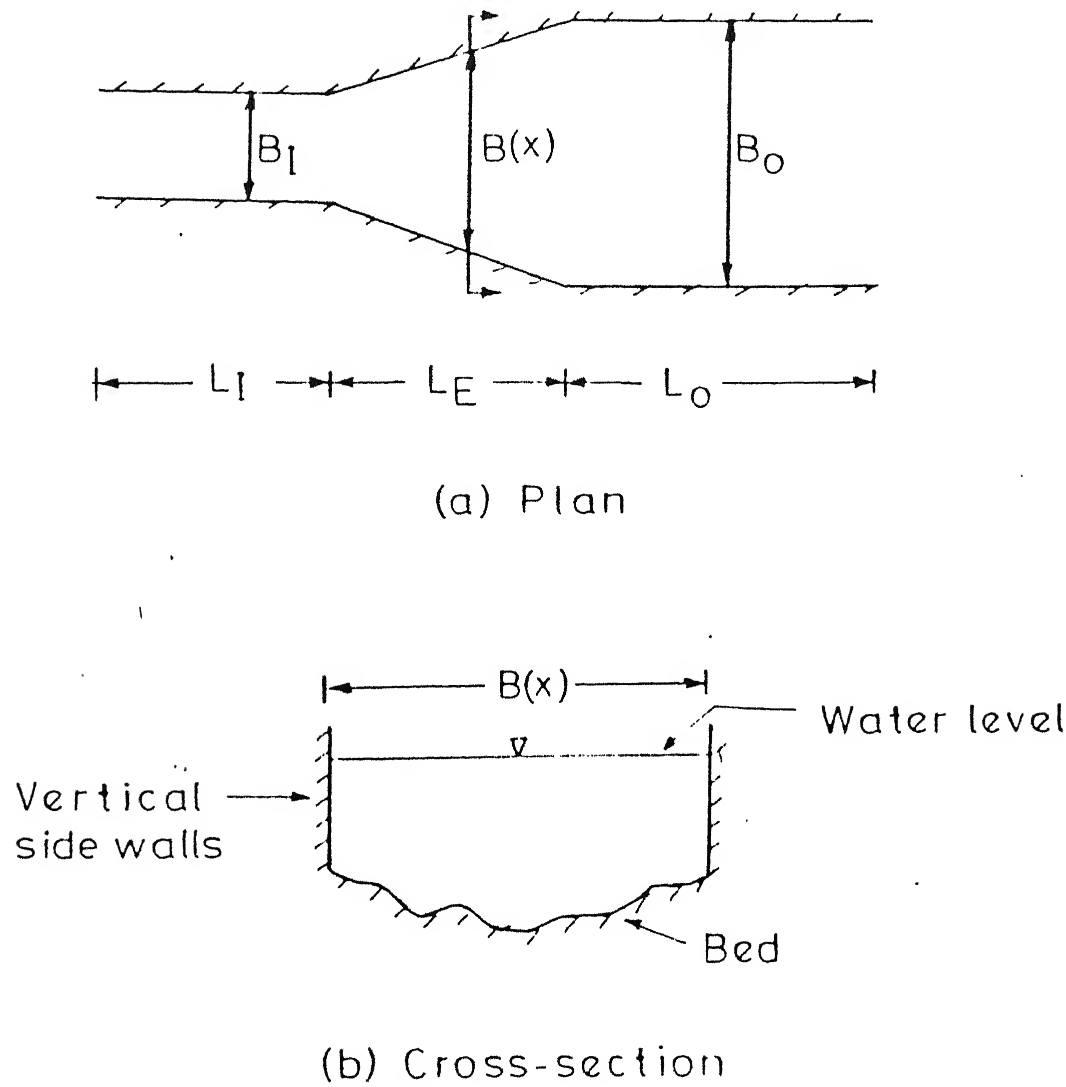


FIGURE 5·1 CHANNEL EXPANSION

(models 1, 2 and 3) and then, the two-dimensional numerical results are compared with the one-dimensional results (models 2,4 and 5).

The analytical as well as numerical results have been obtained for the following input values. Figure 5.1 shows the plan and cross-section of the channel considered. In this figure,

L_I = ength of inlet portion = 5.6 m,

L_E = length of expansion = 4.0 m,

L_O = length of outlet portion = 6.4 m,

BI = channel width at the inlet,

BO = channel width at the outlet = 0.9 m and

B(x) = channel width at any x.

Although the numerical models can be applied to any type of expansion, only a straight walled expansion is considered here. The inlet channel width is varied from 0.5 to 0.8 m in the numerical experiments. The initial uniform bed slope in the downstream direction, SO_x = 0.001 while the transverse slope, SO = 0.0. The downstream control depth, h_d = 0.107 m and flow discharge, Q = 0.019 m^3/s. The sediment transport parameters, a_1 = 0.000145 and a_2 = 5.0 . The bed level at the upstream end, ZI = 0.0 and the porosity of channel bed, λ = 0.4 . The above geometric, flow and sediment properties roughly correspond to the experimental set up of Nashta(1984) for sudden expansion with movable bed.

The numerical models used a finite difference grid of $\Delta\xi$ = 0.4 m and $\Delta\eta$ = 0.1 . A Courant number value of 0.85 is used to achieve numerical stability in the false transient method of

solving water flow equations. The computational time step, Δt for the solution of sediment continuity equation is 1000 seconds. However, Δt has been varied upto 5000 seconds to study its effect on numerical stability and accuracy.

5.1.2 One-dimensional Models

Figures 5.2 to 5.4 show the variation of flow depth,bed level and deposition obtained by using one-dimensional models for BI = 0.8 m (β= $1^{\circ}26'$) and BI = 0.6 m (β= $4^{\circ}18'$).The flow depths for models 1 and 2 (figure 5.2) are matching while model 3 underpredicts depths for small β values.However, the predictions are satisfactory for higher β values or lower BI values (figure 5.2 b). Results from models 1 and 2 are matching very well because of small value of β, in which case the radial and rectangular flow assumptions are equivalent. The error in predictions by model 3 may be due to the numerical approximations. However, the error is not significant and ammounts to less than one percent .

The variations in the bed levels along the channel for the final equilibrium state are shown in figure 5.3.It can be observed that the numerical results (model 3) compare satisfactorily with the analytical results (model 1 and 2).As before, the comparision is better for larger values of β ($\simeq 4^{\circ}$) than for smaller values of β ($\simeq 1^{\circ}$). The numerical results vary in a stair case fashion. Similar type of variation can be observed

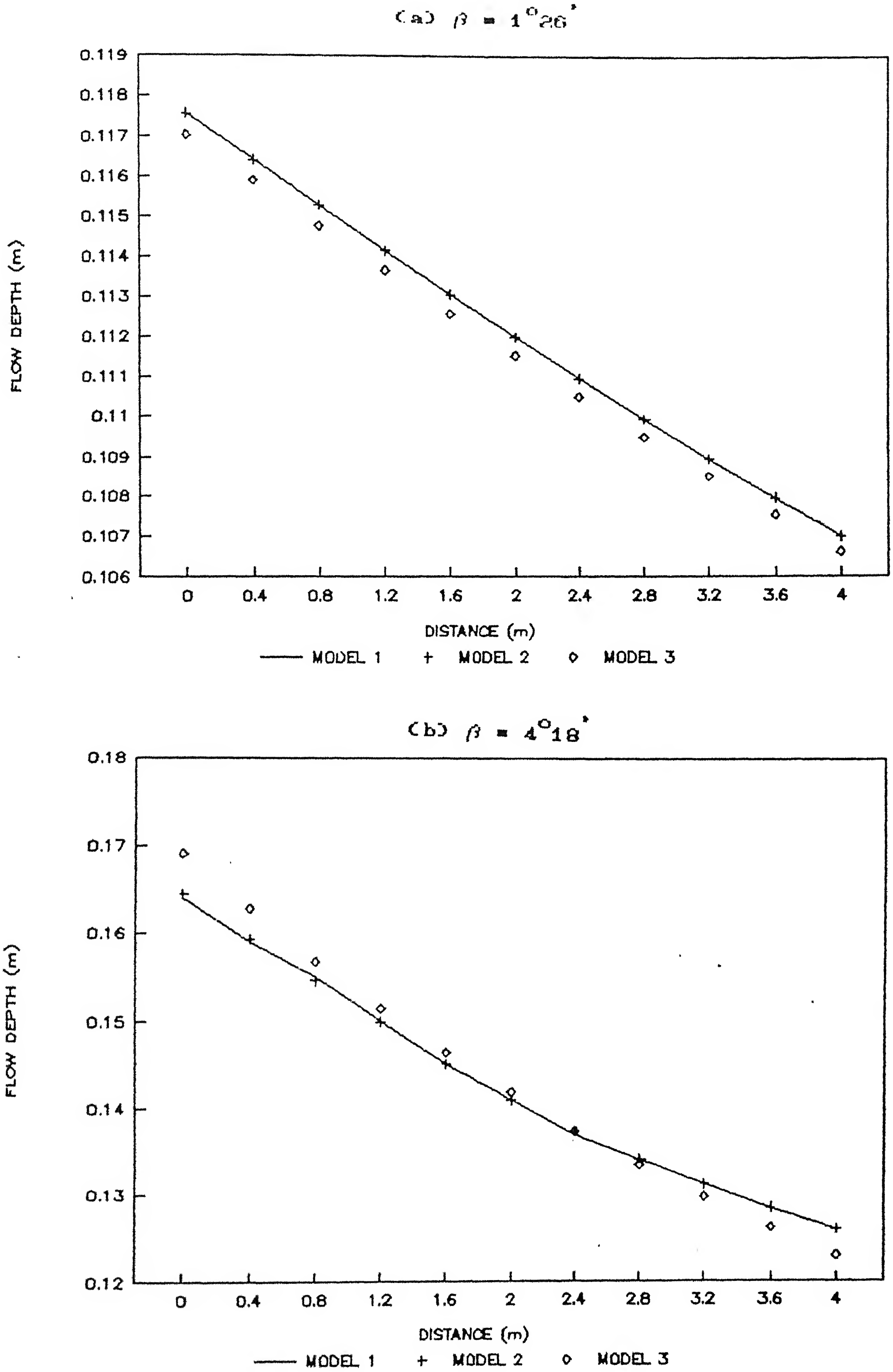
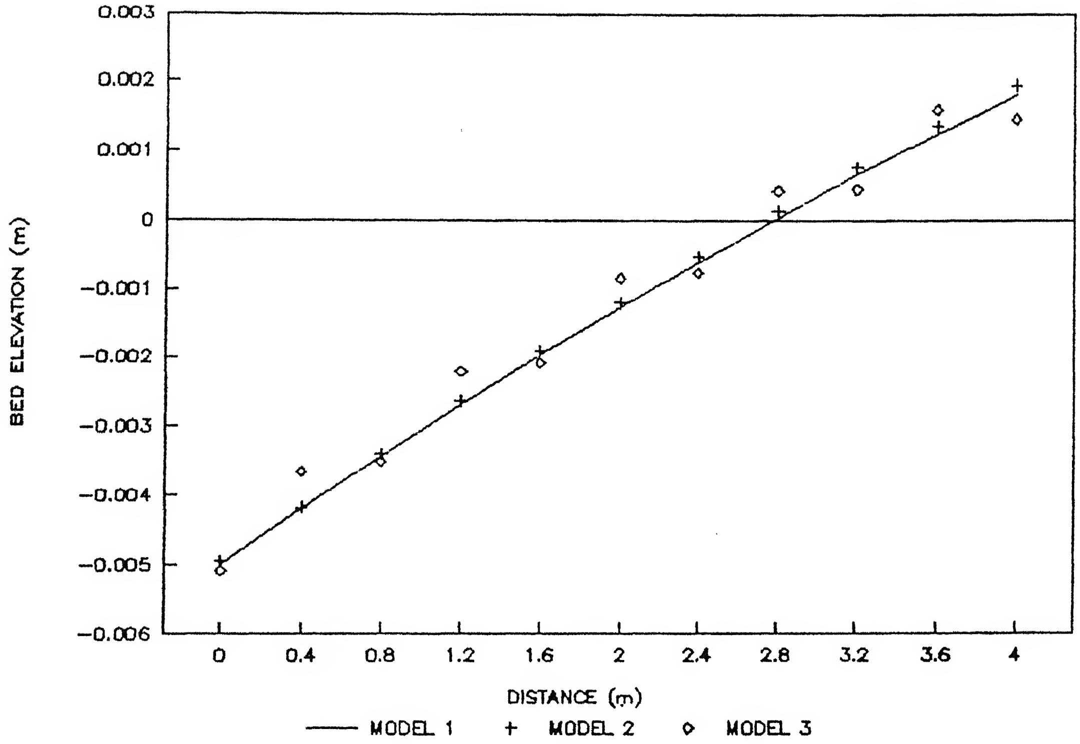


Figure 5.2 Comparison of one-dimensional models for flow depth variation

(a) $\beta = 1^{\circ}26'$



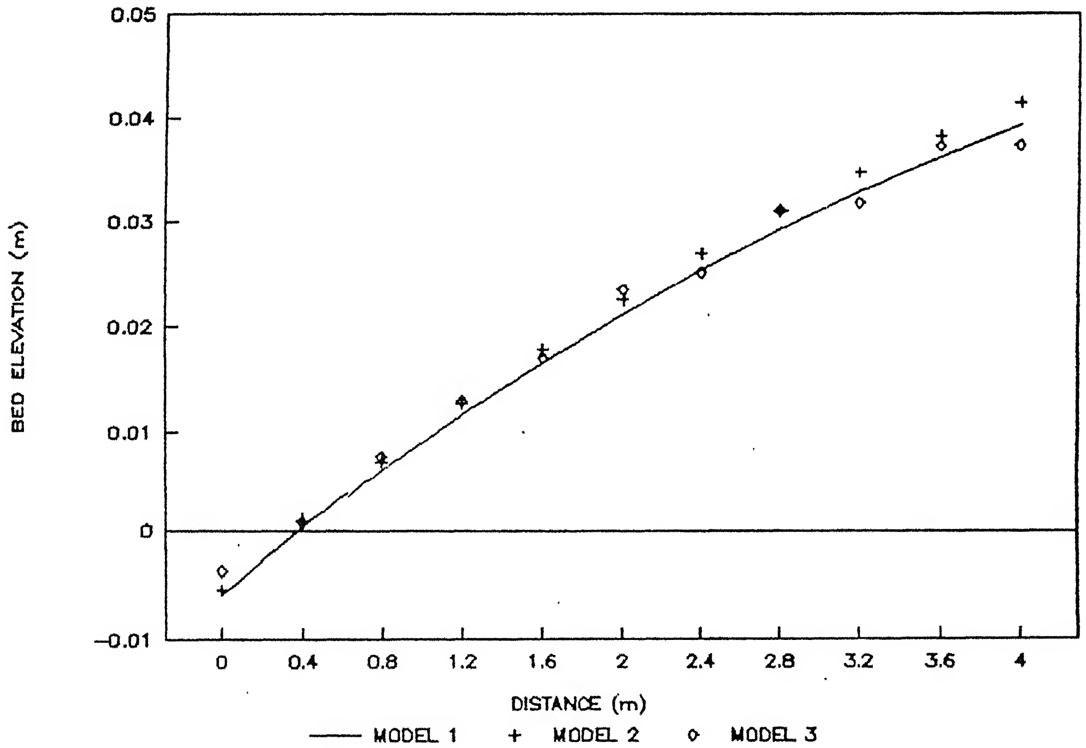
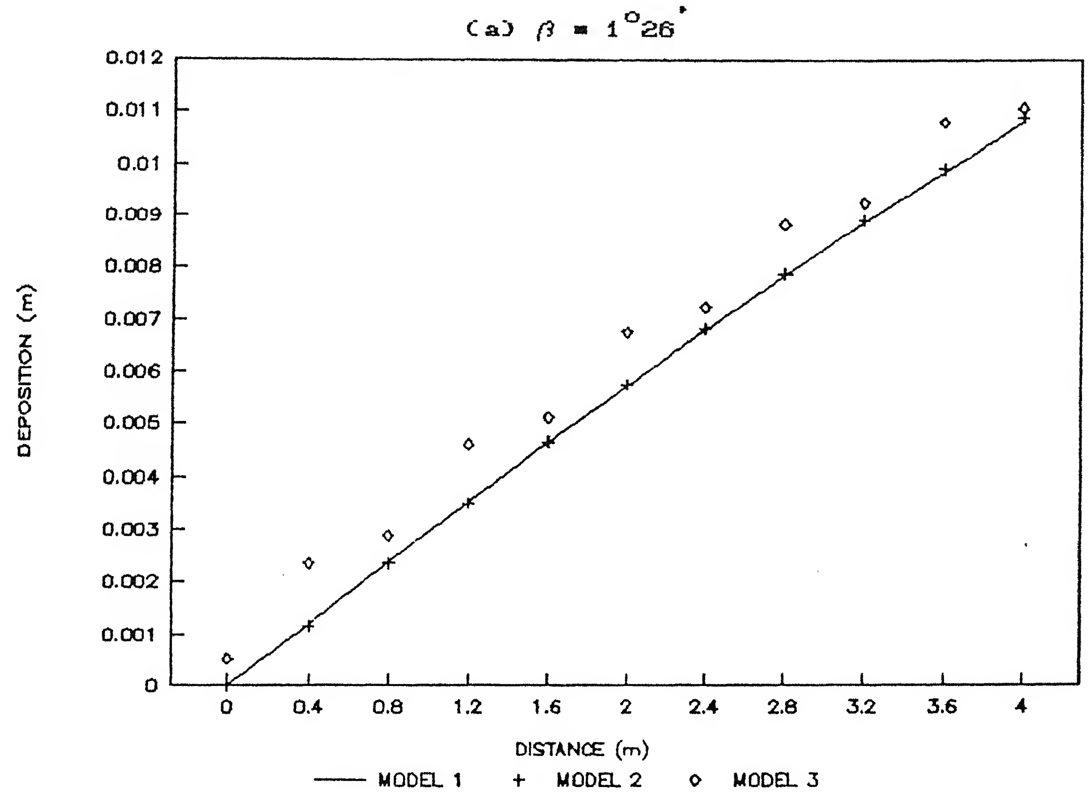


Figure 5.3 Comparison of one-dimensional models for bed level variation



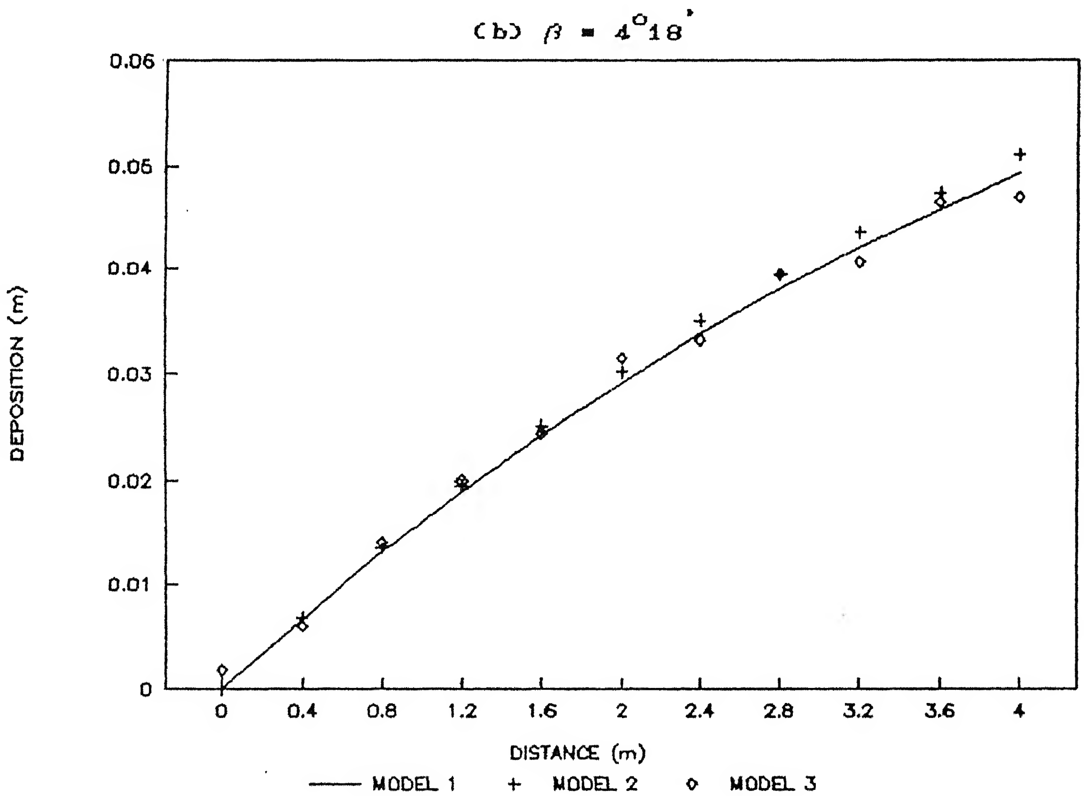


Figure 5.4 Comparison of one-dimensional models for variation of deposition

even in flood routing by Lax-diffusive scheme. This is a characteristic of the finite-difference scheme adopted. It can also be observed that the rectangular flow model (model 2) overpredicts the aggradation as compared to the radial flow model (model 1) for large values of β (figure 5.4 b). However, this difference is not very significant . It should be noted that the results for models 1 and 2 are presented along the channel center line. The actual bed level contours as obtained using radial flow model will differ slightly from those obtained using the rectangular flow model. This is illustrated in figure 5.5.

The above discussion shows that the concept of quasi-steady uncoupled modelling as incorporated in model 3 can be successfully applied to predict the long term bed level variation in gradual expansions. However, model 3 assumes one-dimensional flow conditions and should not be used in situations where flow is highly two-dimensional. In the next section, results obtained using two-dimensional models are presented.

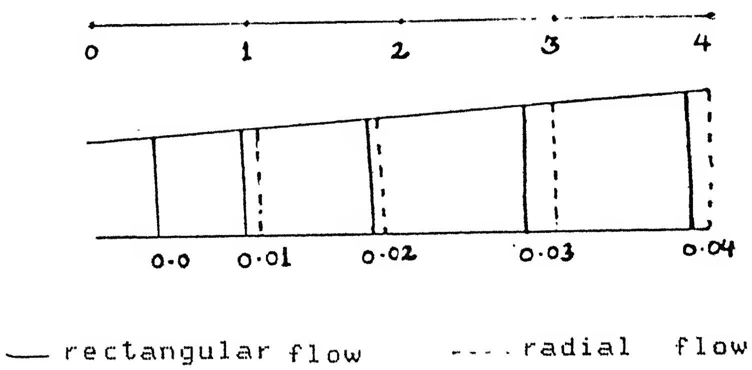


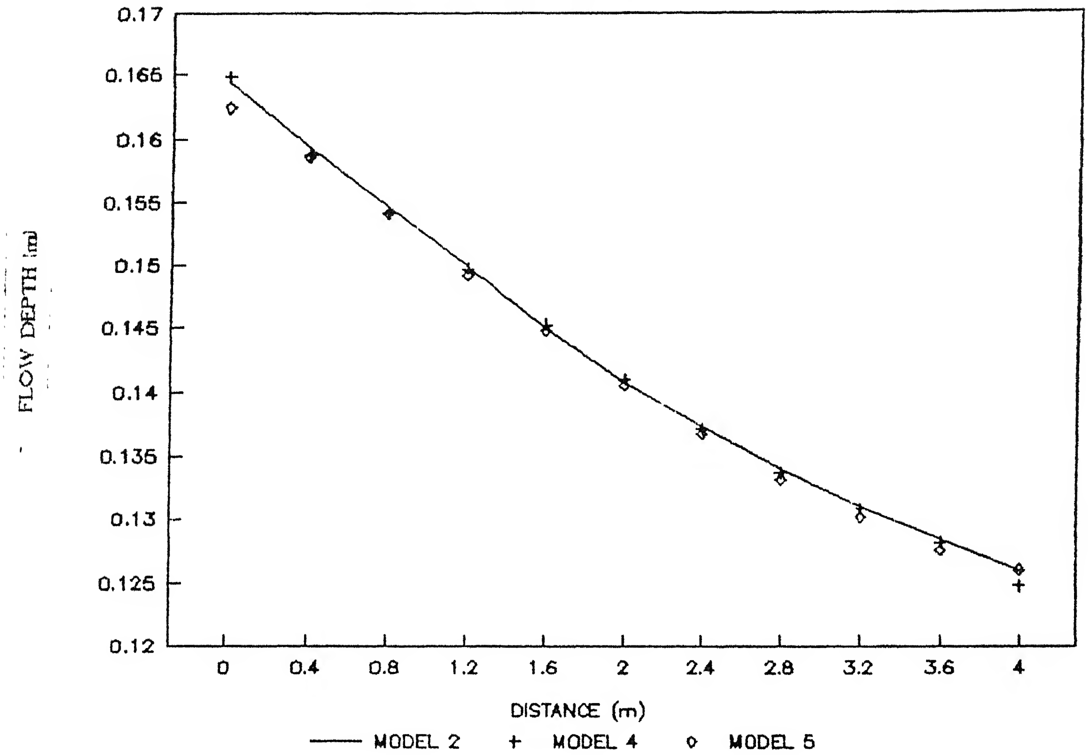
Figure 5.5 Bed level contours

5.1.3 Two-dimensional models

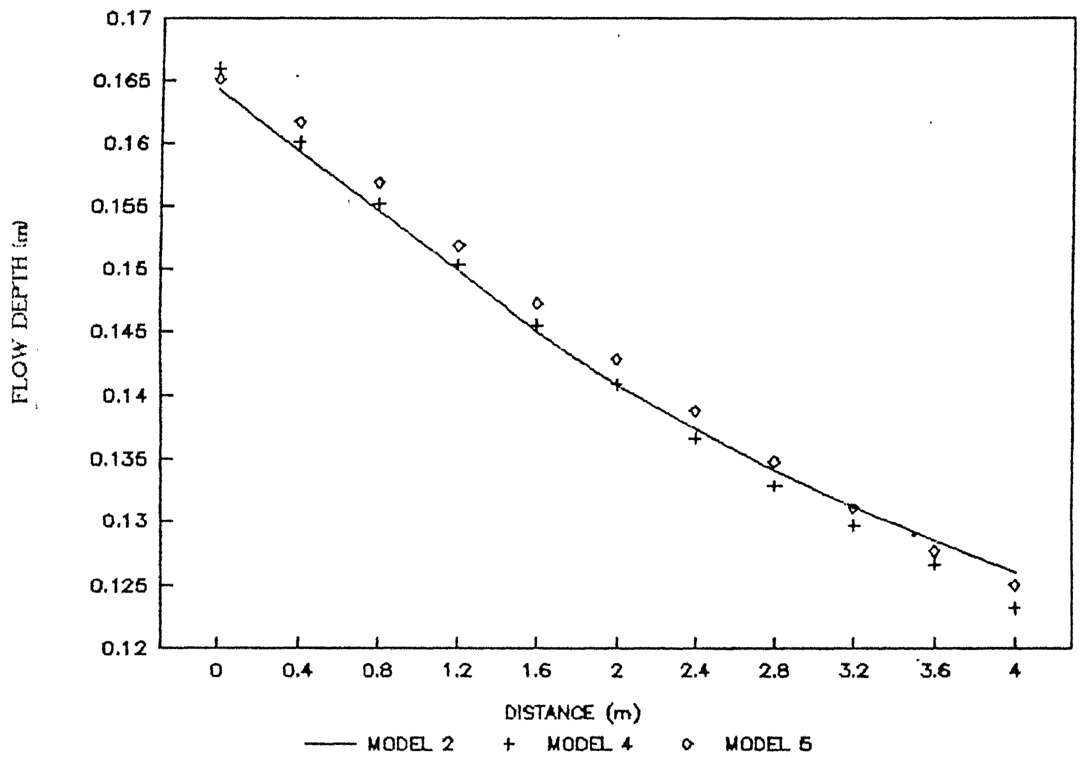
The two-dimensional models (model 4 and model 5) also use the concepts of quasi-steady flow conditions and uncoupling of sediment and flow equations As mentioned earlier, they use the false transient method to obtain the steady flow parameters during any computational time . Model 4 uses Lax-diffusive scheme and model 5 uses MacCormack scheme for this purpose. Two different numerical schemes are used in this study so that there is more confidence in the numerical results.Figure 5.6 - 5.8 show the results obtained using these models. The analytical results obtained using model 2 are also presented in these figures for the purpose of comparision.As before, all the computations are made for two different expansion angles, $\beta = 1^\circ 26'$ and $\beta = 4^\circ 18'$. The comparisions are made for variation along the centerline as well as along the wall for the final equilibrium state.

Figure 5.6 shows the variation of flow depth with distance . As can be observed from these figures, the numerical results from models 4 and 5 compare very well with each other. The numerical results also compare satisfactorily with the analytical results obtained using model 2. Although flow depths obtained by one-dimensional model are generally higher than those predicted by two-dimensional models (figure 5.6), this difference is not significant. Figure 5.7 shows the variation of bed levels with distance for the final equilibrium state. As before, the numerical

(i) Along wall



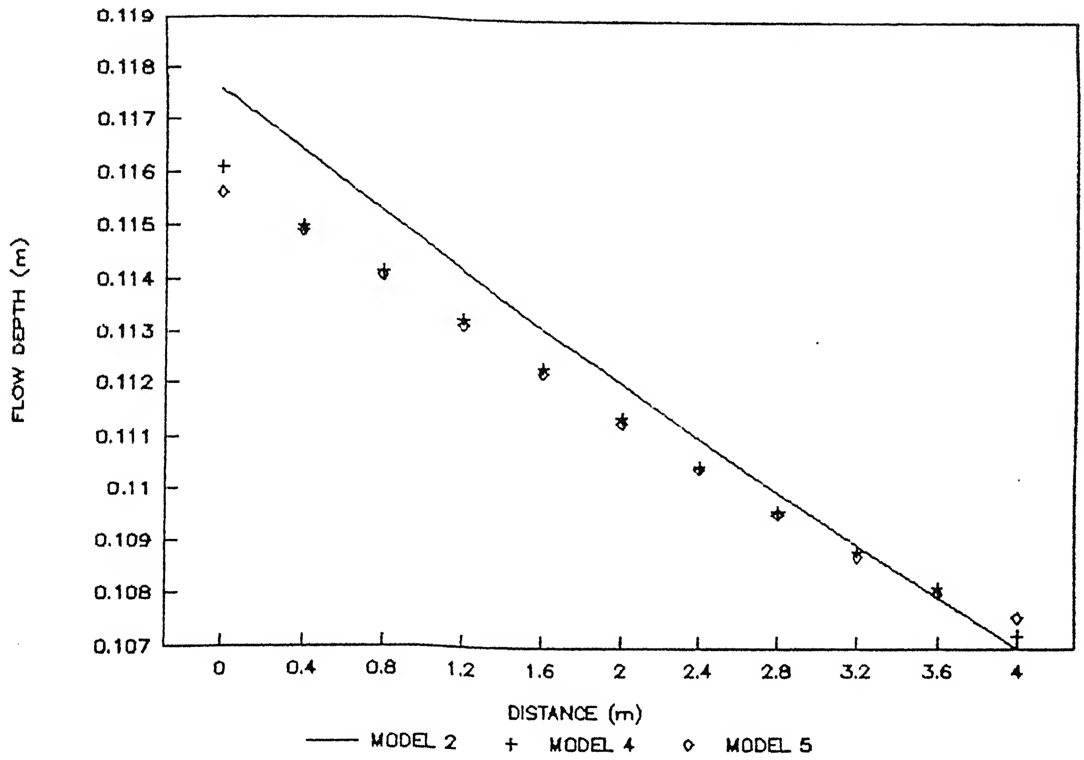
(ii) Along center line



(a) $\beta = 4^{\circ}18'$

Figure 5.6 Comparison of two-dimensional models for flow depth variation

(i) Along wall



(ii) Along center line

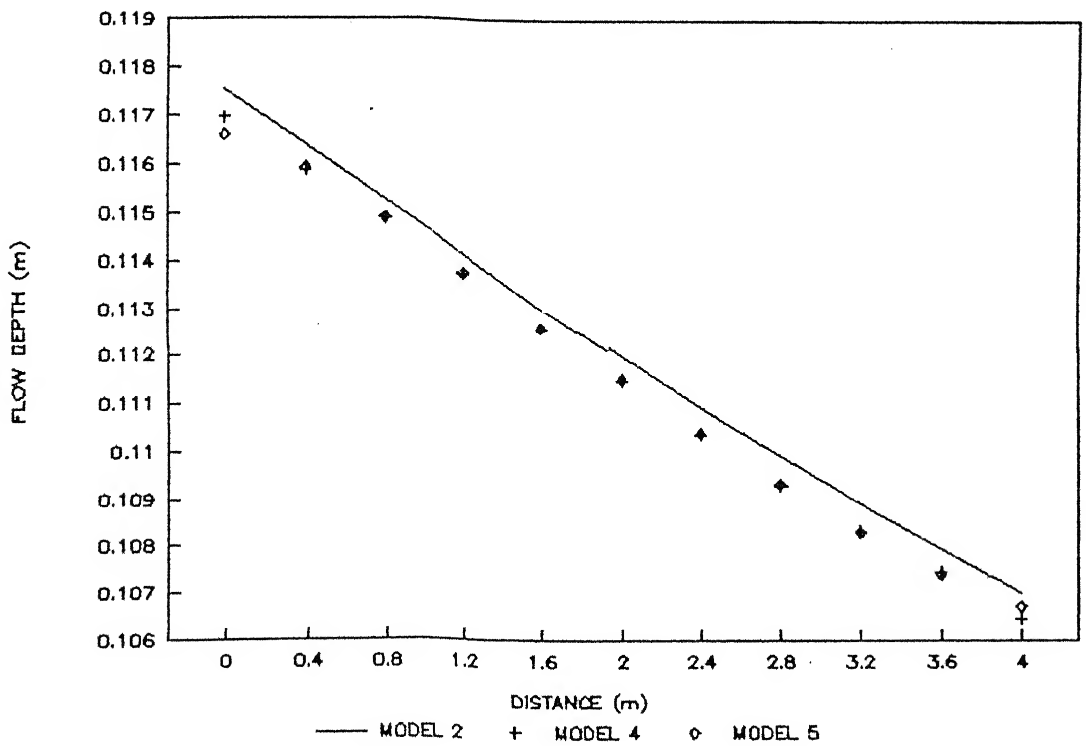
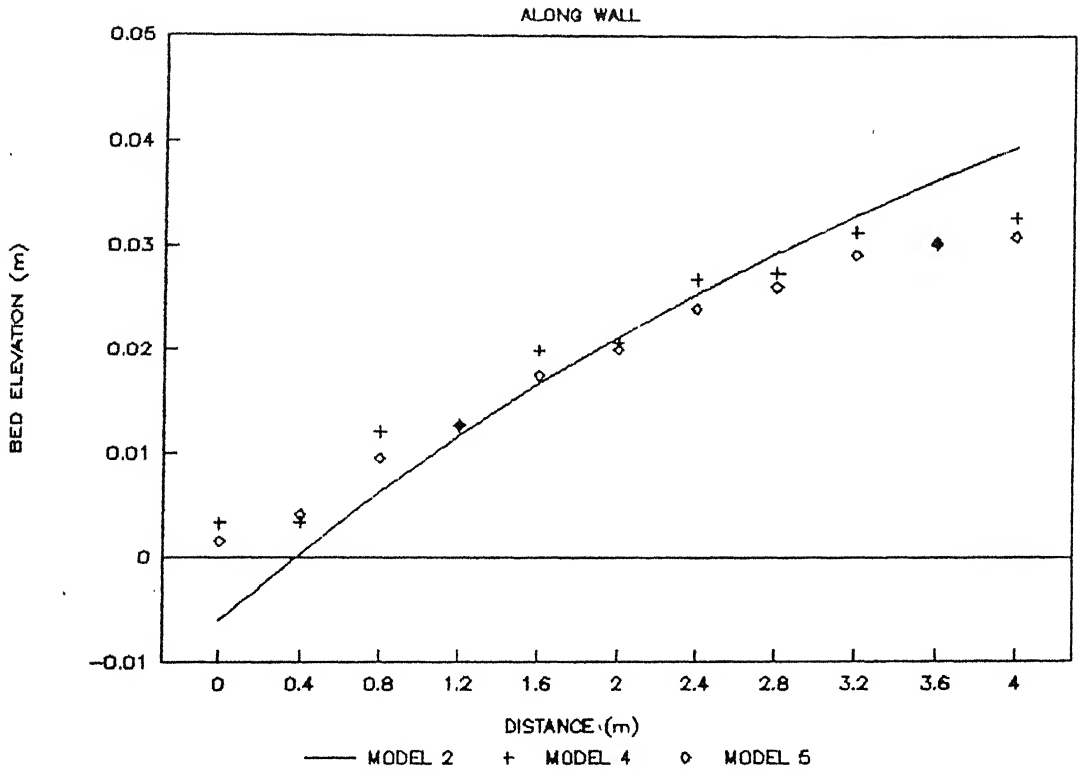
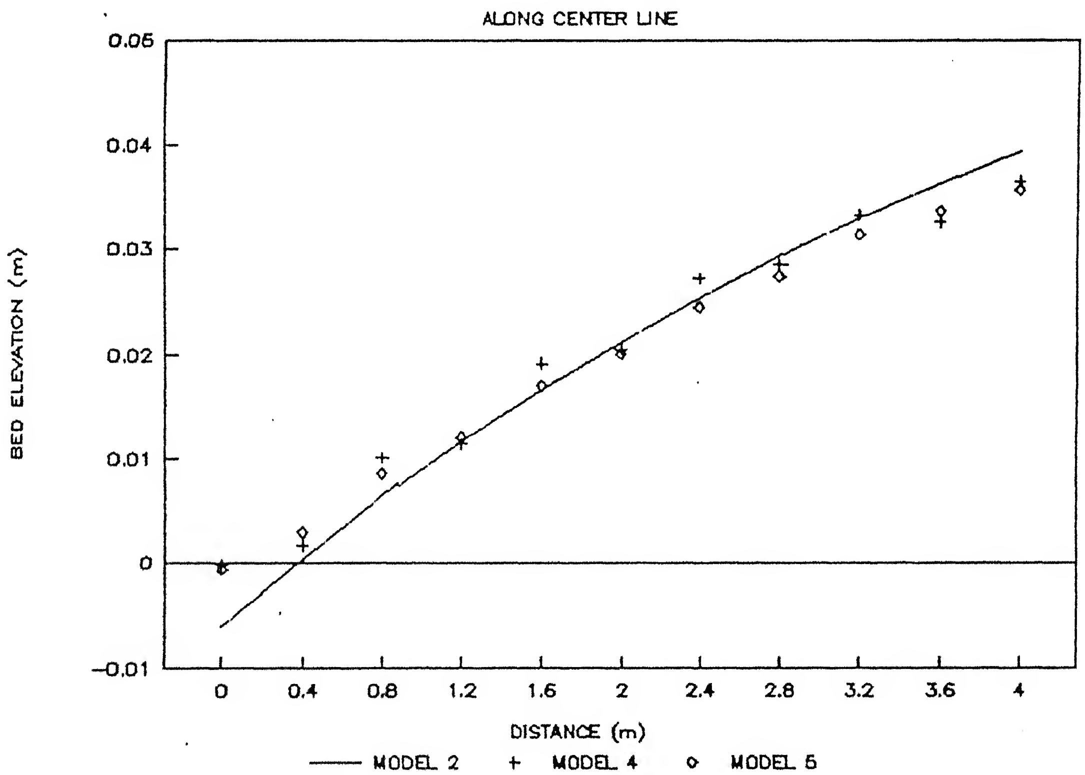


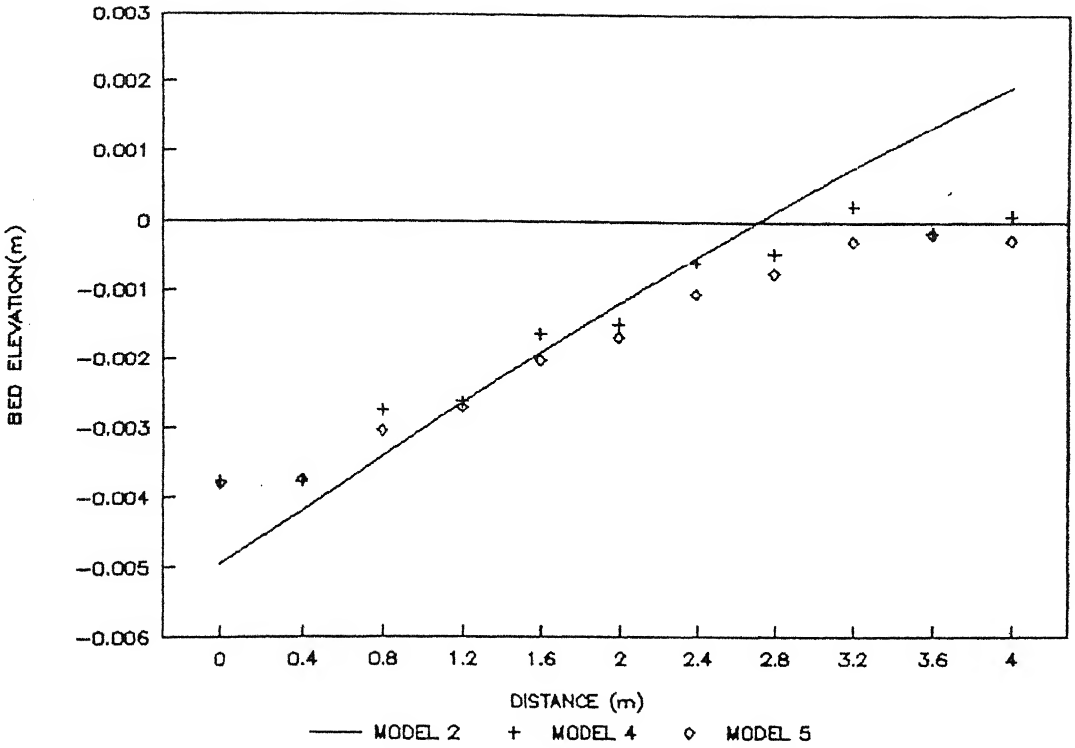
Figure 5.6 cont. (b) $\beta = 1^{\circ}26'$





(a) $\beta = 4^{o}18'$

Figure 5.7 Comparison of two-dimensional models for bed level variation



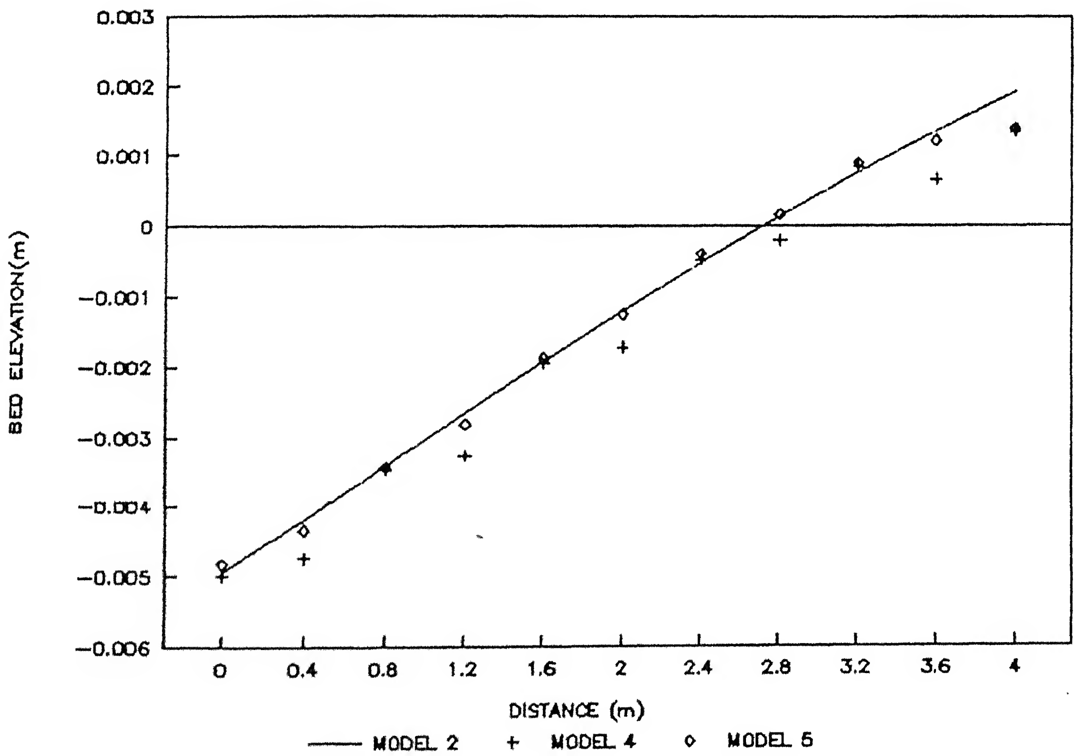
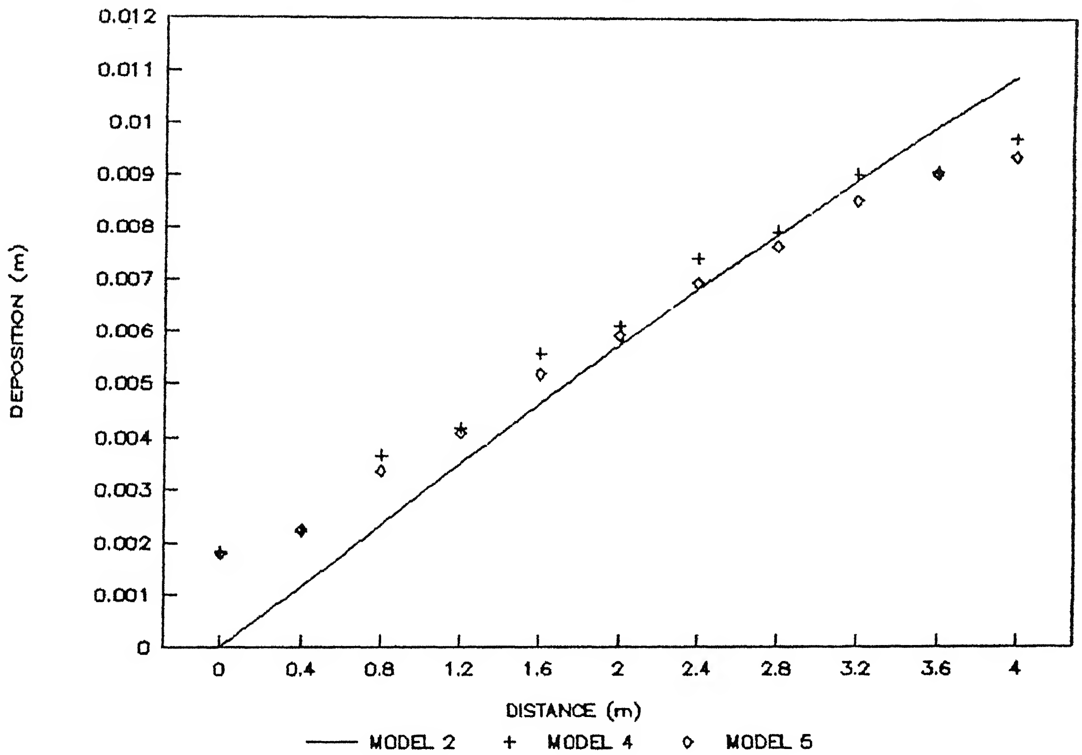
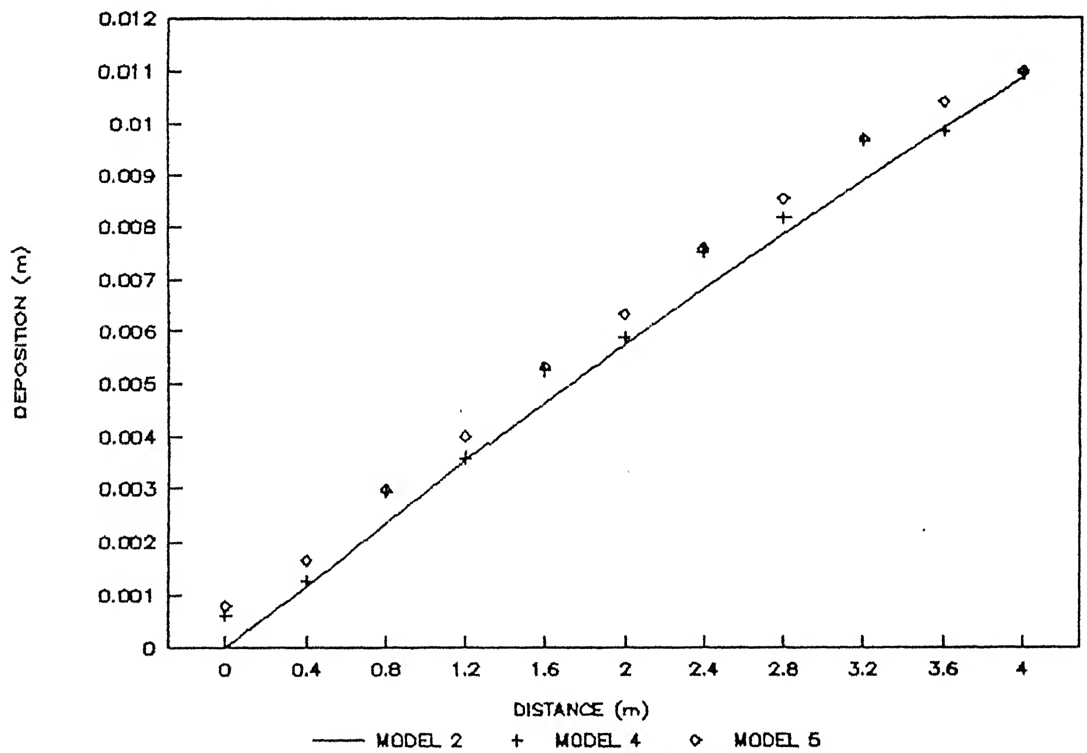


Figure 5.7 cont. (b) $\beta = 1^{o}26'$

(i) Along wall



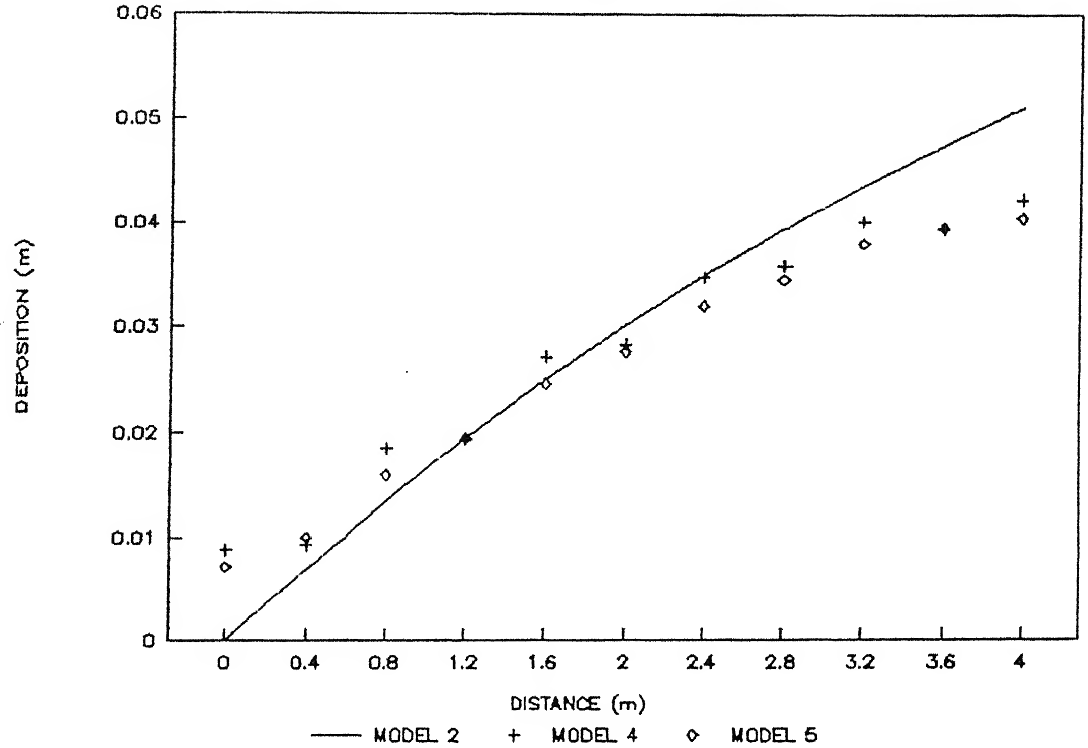
(ii) Along center line



(a) $\beta = 1^{o}26'$

Figure 5.8 Comparison of two-dimensional models for variation of deposition

Along wall



Along center line

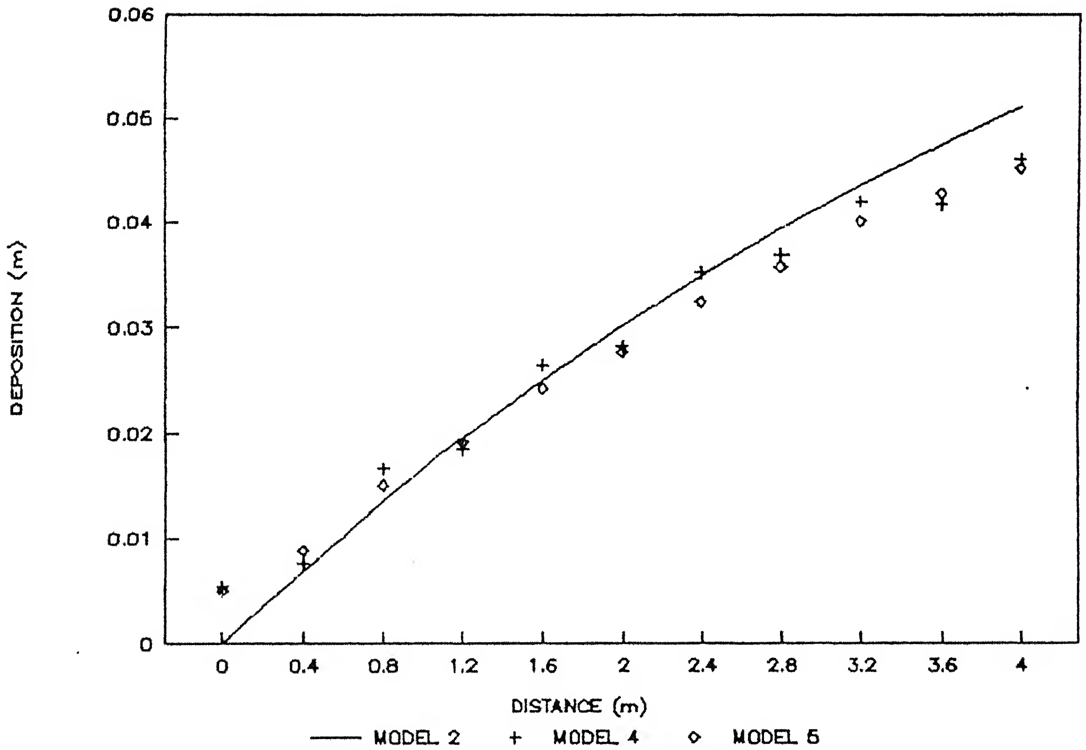


Figure 5.8 cont. (b) $\beta = 4^{\circ}18'$

results obtained using model 4 match well with those obtained using model 5 giving certain amount of confidence in our numerical results. The stair case type variation can be observed in the case of model 4 which uses Lax-diffusive Scheme. This is eliminated in case of model 5 which uses MacCormack scheme It should be noted that MacCormack Scheme is second order accurate while Lax-diffusive scheme is only first order accurate. The two-dimensional effects in aggradation can be observed from figure 5.7 . Considering the deposition along the wall,the bed levels obtained using models 4 and 5 are higher than the bed levels obtained using the one-dimensional model in the initial reaches of the expansion. As a consequence, the two-dimensional models give lower bed levels than those obtained by one-dimensional model towards the outlet of the expansion. However, this difference is not significant along the center line, especially for the smaller β values (figure 5.7 a-ii). It can also be observed that, as expected, the two-dimensional effects become stronger when the expansion angle (β)is large . Figure 5.8 shows the variation of deposition with distance. Similar trends are observed as before. Although the numerical results obtained using the two-dimensional models differ from those obtained using one-dimensional model, they are of the same order of magnitude. This illustrates the satisfactory performance of two-dimensional numerical models.

5.2 SENSITIVITY ANALYSIS

The developement of any mathematical model is not complete untill a sensitivity analysis is performed for dependence

on numerical parameters. In this section, the sensitivity of numerical results to grid size and computational time step are discussed. The results are presented for the same flow and sediment data as before. Although computations are done for both $\beta = 1^\circ 26'$ and for $\beta = 4^\circ 28'$, results for only $\beta = 1^\circ 26'$ are presented. Similar results are obtained for $\beta = 4^\circ 28'$. The numerical results obtained using model 4 as well as model 5 are considered.

5.2.1 Effect of Grid Size

Figure 5.9 shows the effect of computational distance step in downstream direction, DS on the deposition for model 4. As can be seen, the difference in the numerical results for the three values of DS = 0.2, 0.4 and 0.5 is not significant, especially for deposition along the center line. However, this insensitivity is not as good for deposition along the wall. This may be due to the reflection boundary technique used for simulating the side wall boundary conditions. This technique is only approximate and unfortunately, no other alternative method is available. Similar observations can be made for numerical results obtained using model 5 (figure 5.10). Figures 5.11 and 5.12 show the sensitivity of numerical results to the value of computational distance step in transeverse direction, DN for models 4 and 5 respectively. Three different values of DN equal to 0.833, 0.1 and 0.125 are used in the computations. Again, same ~~same~~ trends as discussed above are observed.

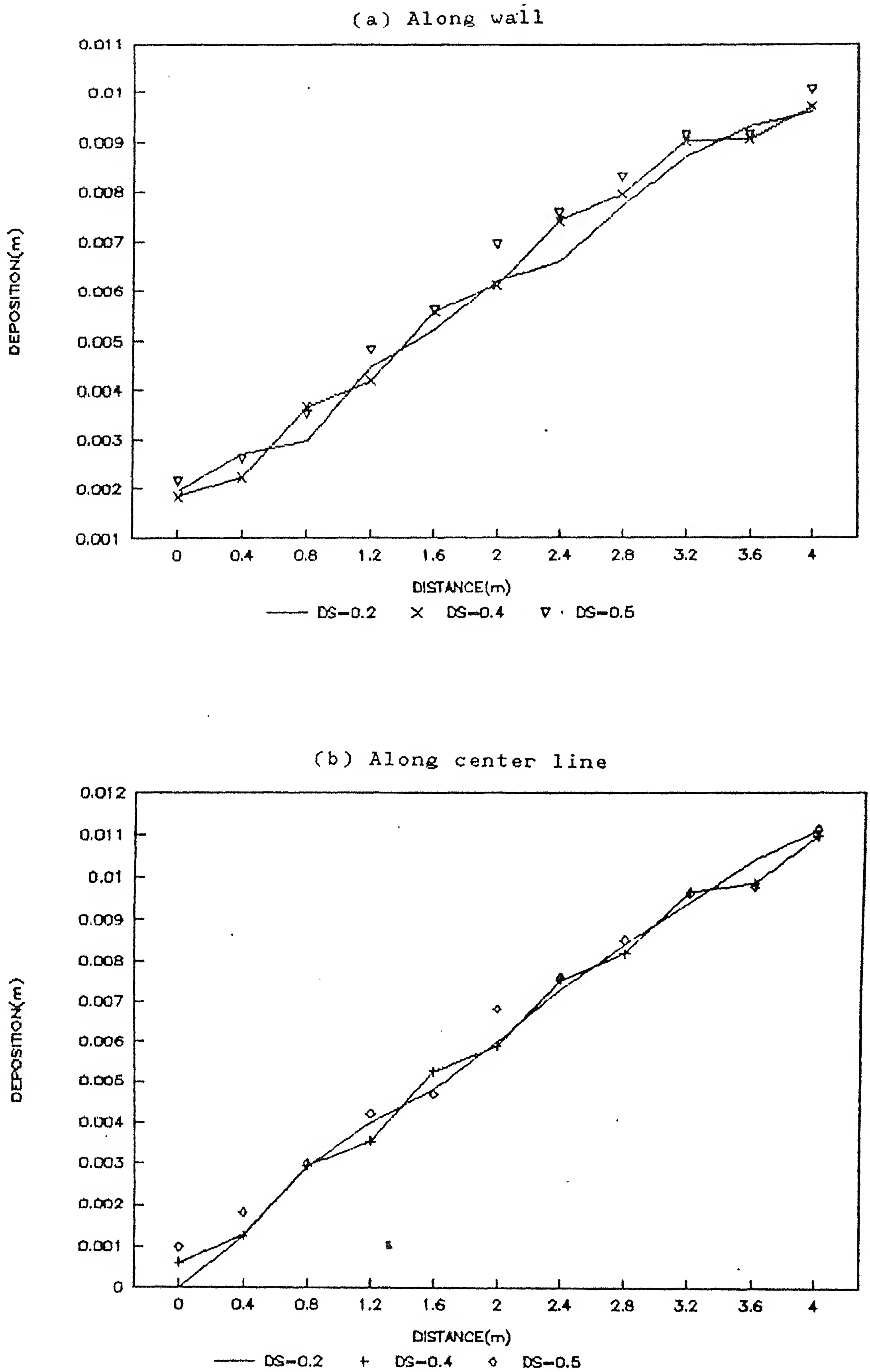
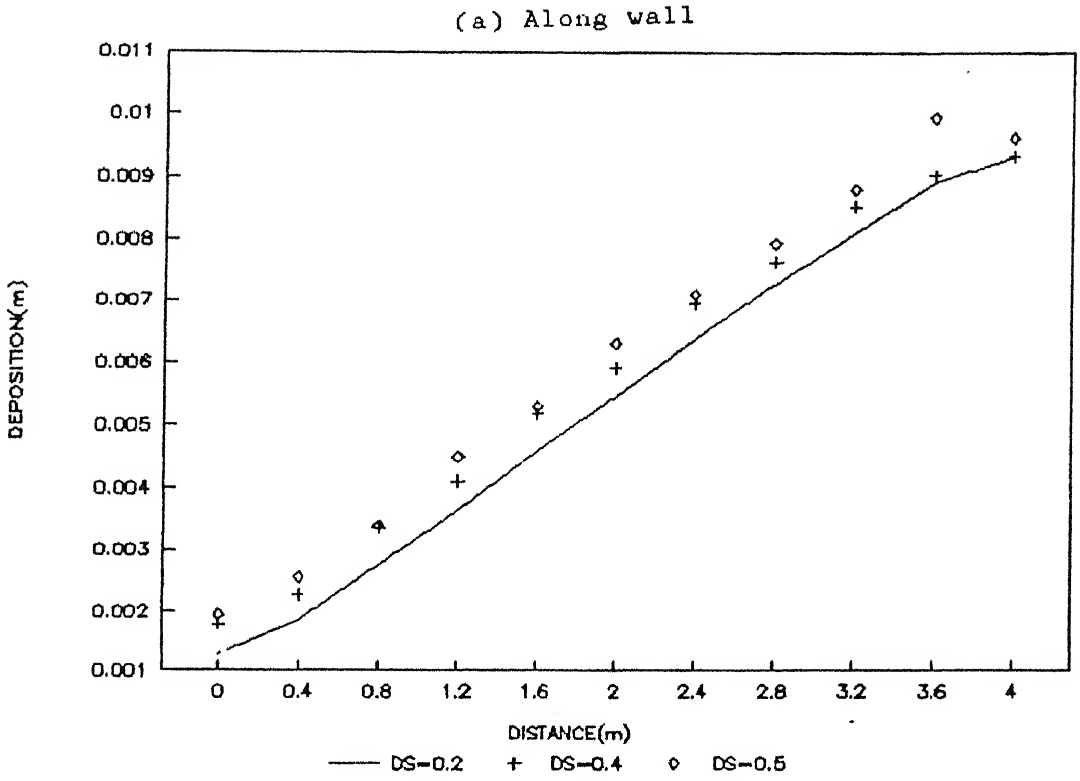


Figure 5.9 Variation of deposition for different values of DS

(using Lax scheme in flow calculations)



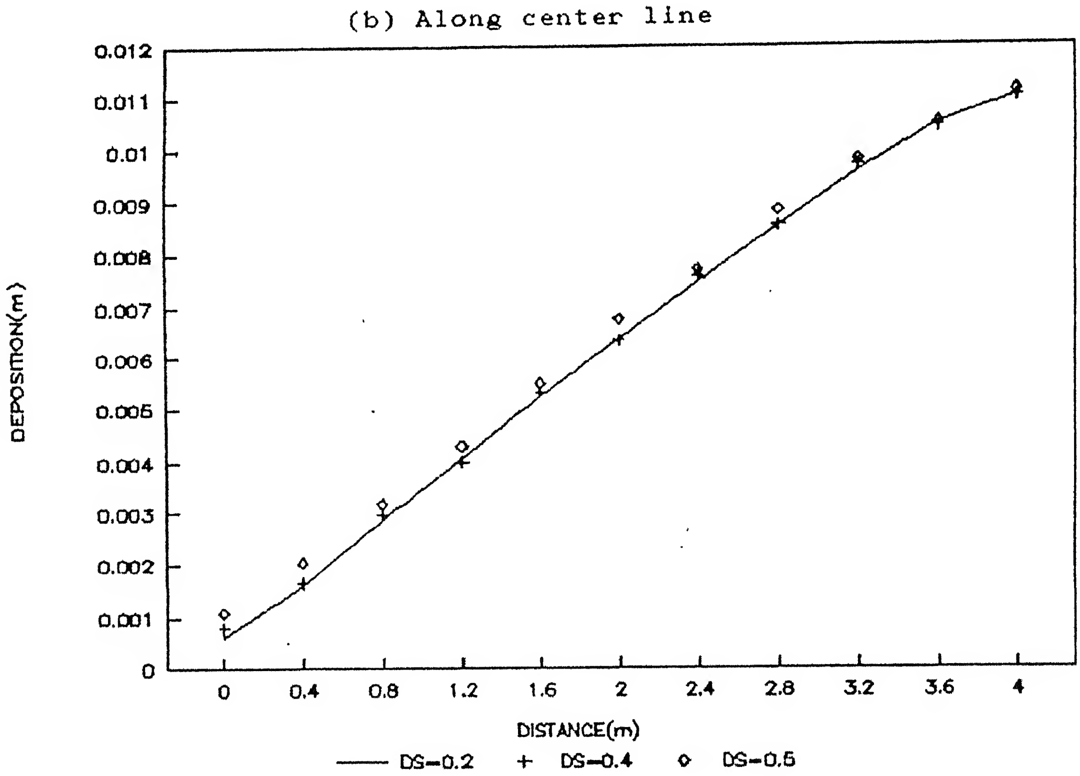
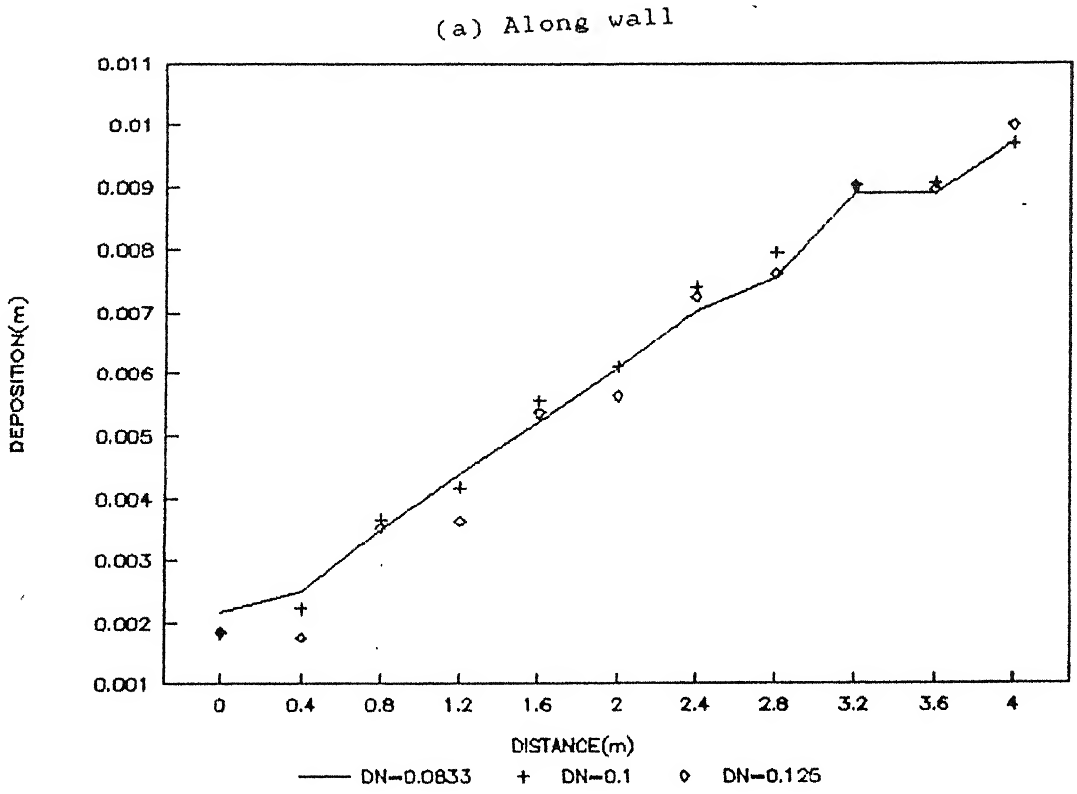


Figure 5.10 Variation of deposition for different values of DS

(using MacCormack scheme for flow calculations)



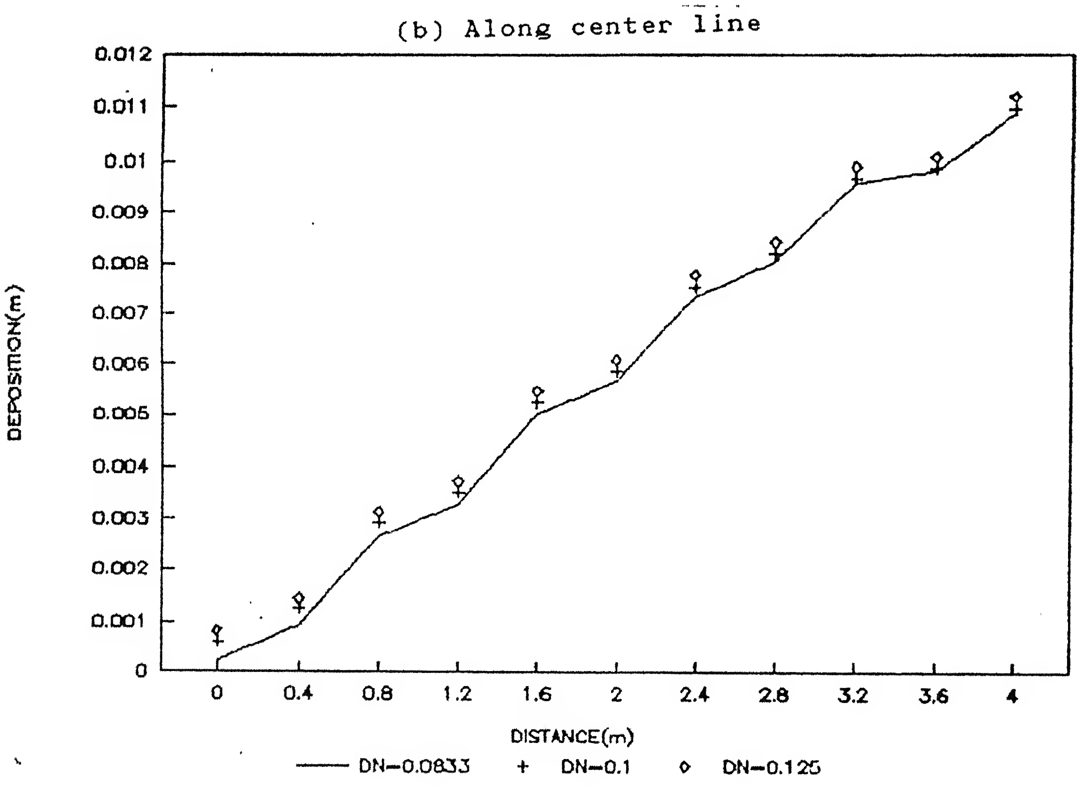
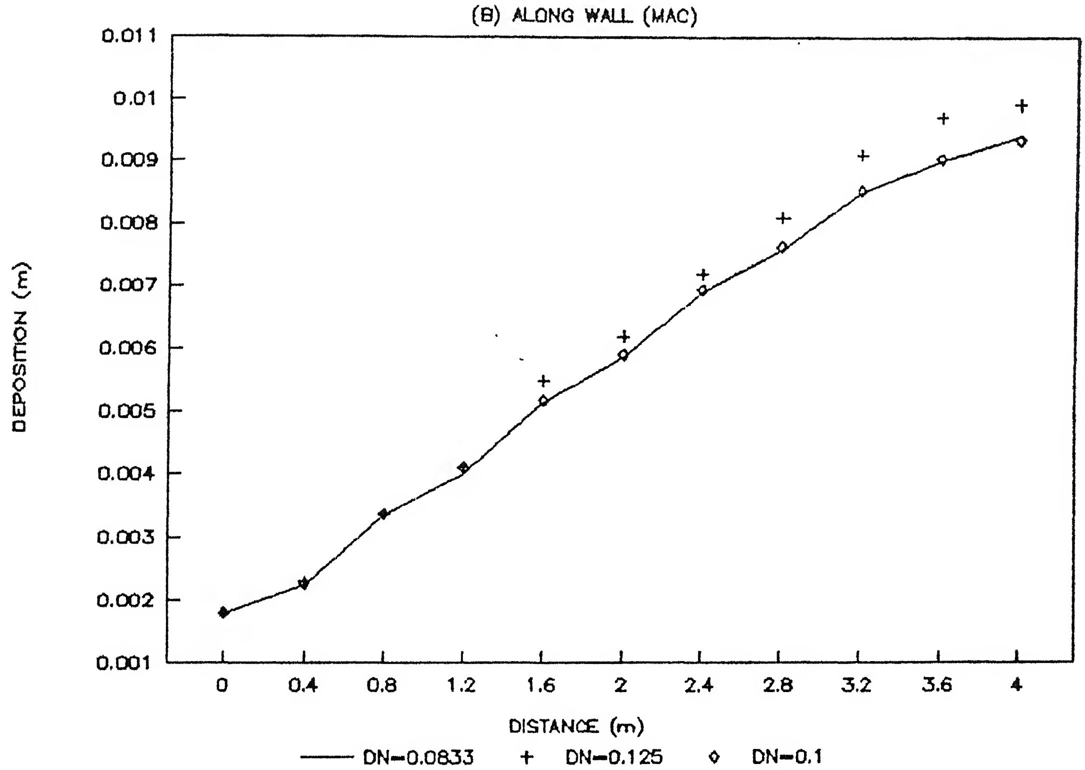


Figure 5.11 Variation of deposition for different values of DN (using Lax scheme for flow calculations)



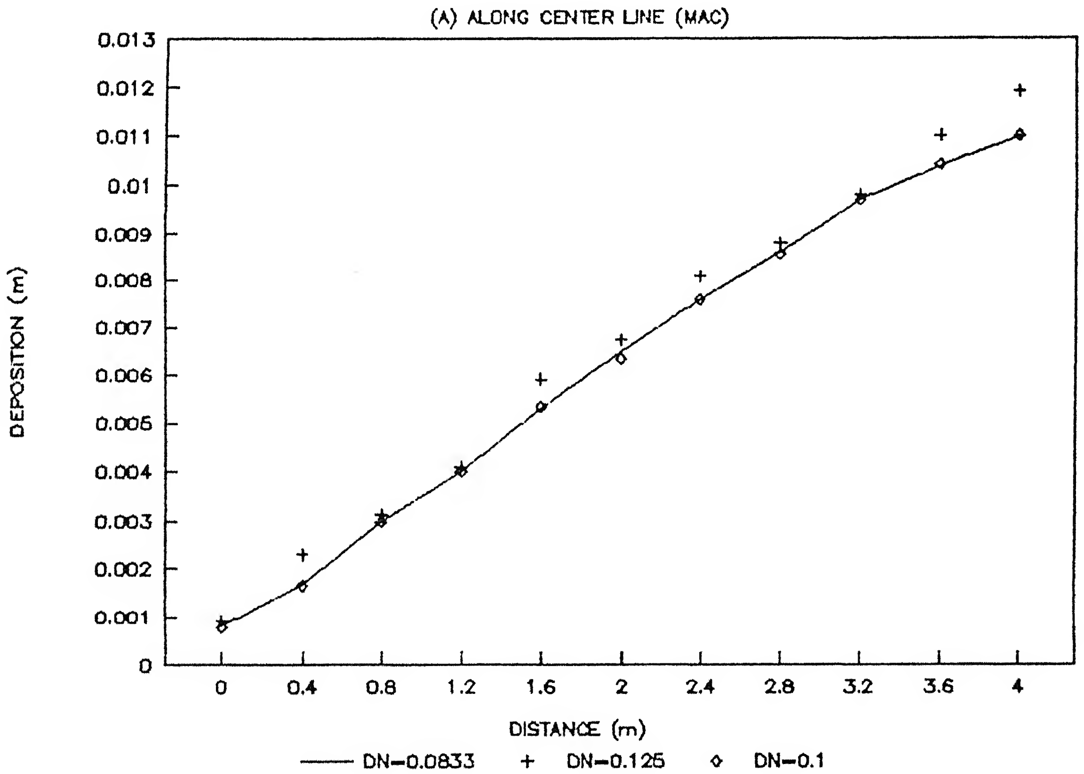


Figure 5.12 Variation of deposition for different values of DN (using MacCormack scheme for flow calculations)

5.2.2 Effect of computational time step

No theoretical guidelines are available for selection of the computational time step, DTIME in quasi-steady , uncoupled models. Therefore, it is necessary to do some numerical experimentation before a proper value is chosen. It should be chosen such that it is not too large so as to introduce instabilities . At the same time , it should not be too small so that the computational cost is not prohibitive. Figures 5.13 and 5.14 show the effect of computational time step on numerical results for models 4 and 5 respectively. Three different values of DTIME equal to 1000 s, 2000 s and 5000 s are used. As can be observed from these figures, all the three DTIME values gave the same results. Although not presented here, an extensive numerical experimentation indicated that the maximum value of DTIME that can be used without stability problems strongly depends on the expansion angle as well as sediment parameters. For example , stable results could not be obtained using model 5 for $\beta = 4^{\circ}18'$ when DTIME was greater than 2000 s . The results presented here indicate that once we get stable results with a particular DTIME value, any DTIME value smaller than this would not increase the accuracy significantly.

5.3 PARAMETRIC STUDY

In this section, the effect of different parameters on the final equilibrium state is studied. Only one parameter

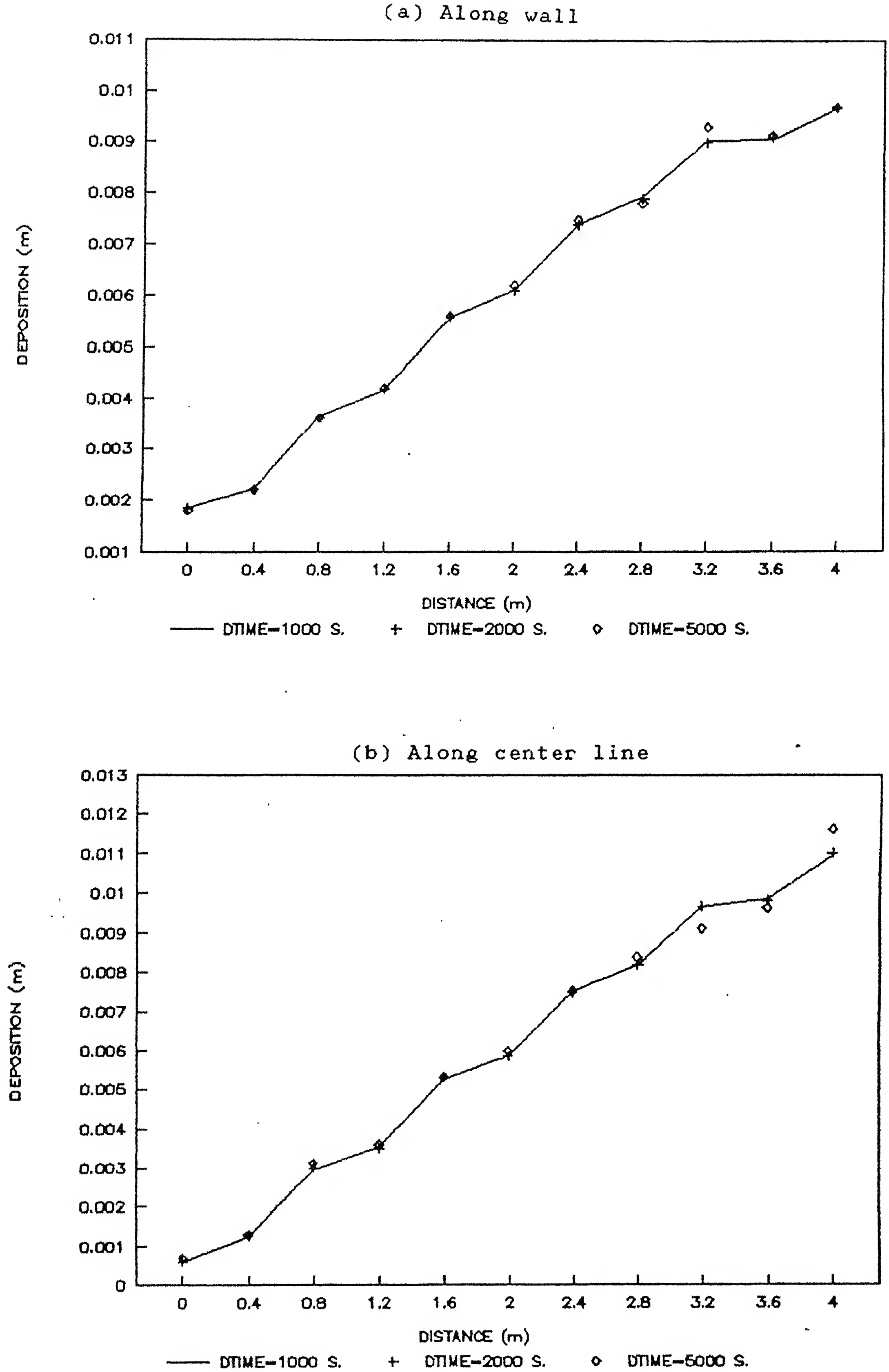
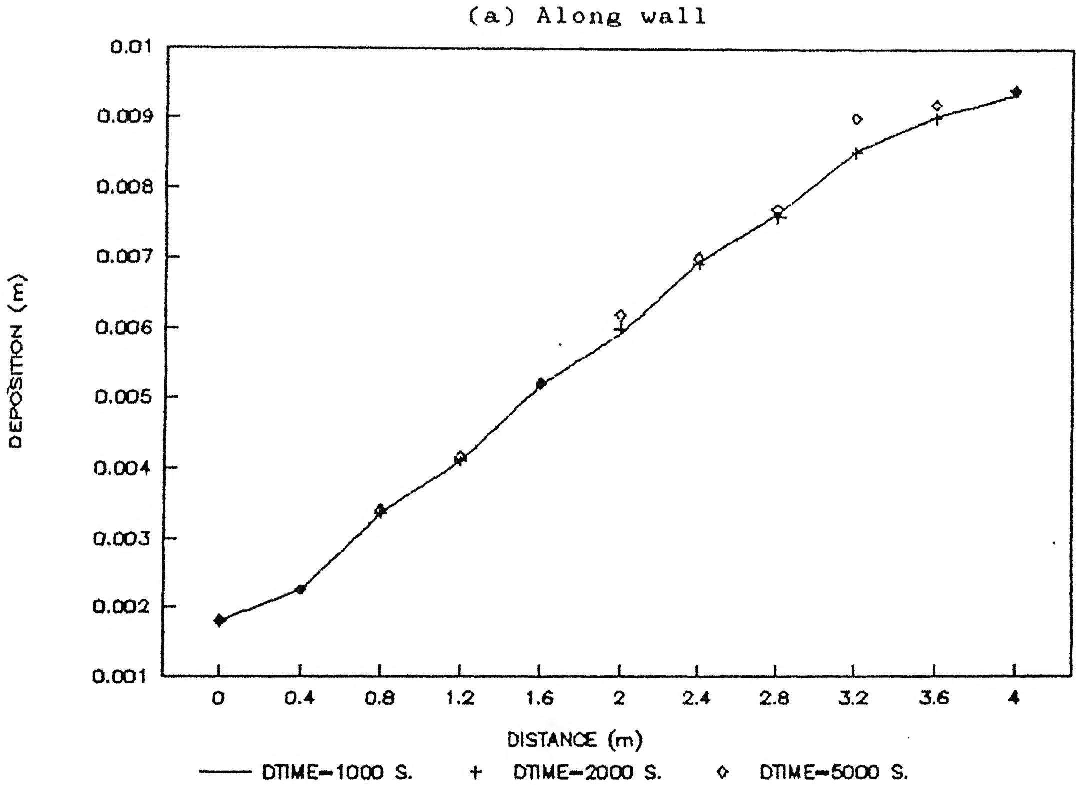


Figure 5.13 Variation of deposition for different values of DTIME (using Lax scheme for flow calculations)



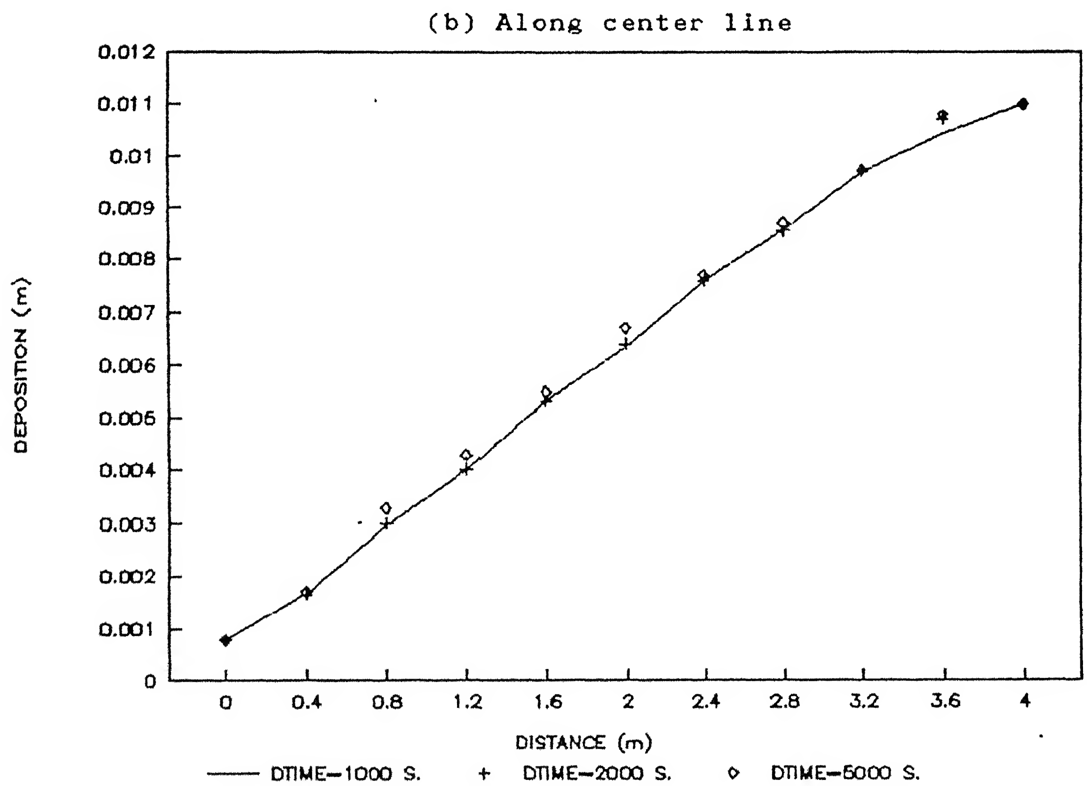


Figure 5.14 Variation of deposition for different values of DTIME

(using MacCormack scheme for flow calculations)

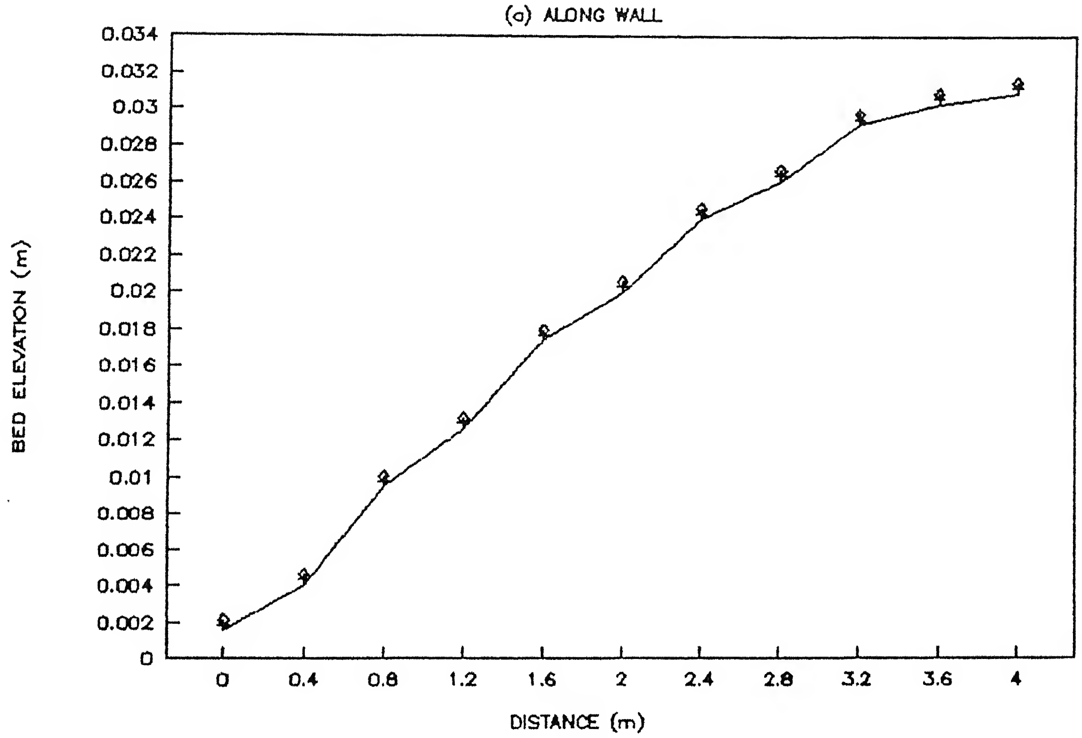
is varied at a time and others are kept constant. Table 5.1 shows the several cases studied

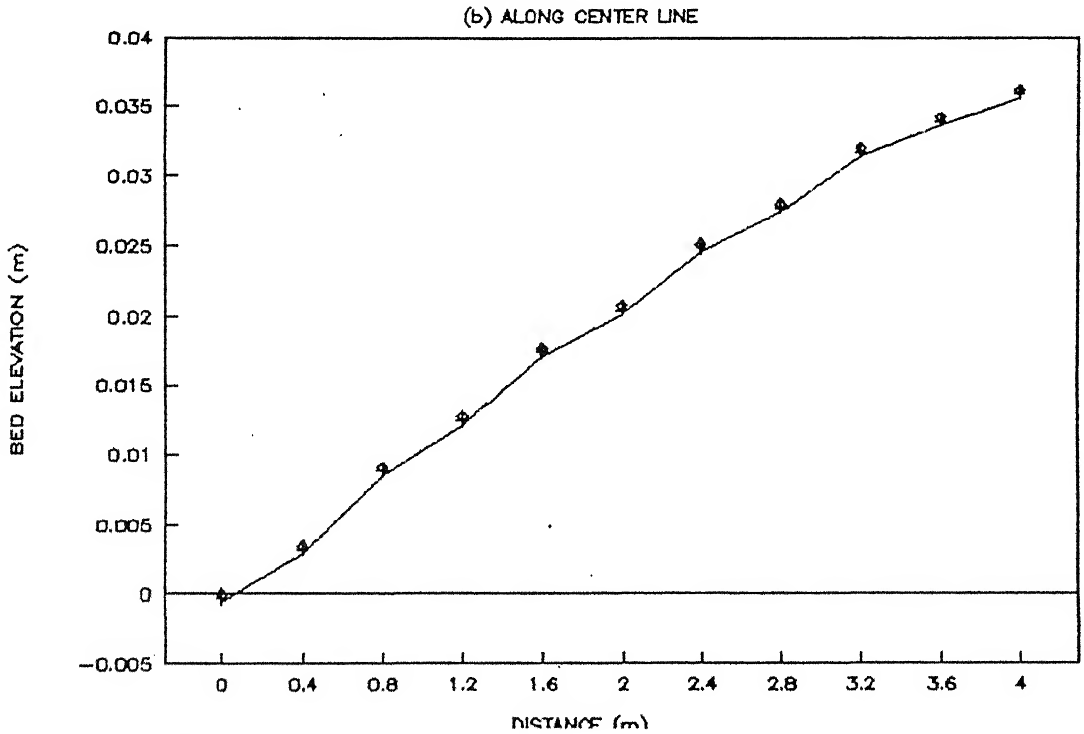
TABLE 5.1

Sl. No.	BI	a_1	a_2	h_d
1	0.8	0.000145	5.0	0.107
2	0.7	0.000145	5.0	0.107
3	0.6	0.000145	5.0	0.107
4	0.6	0.000290	5.0	0.107
5	0.6	0.000435	5.0	0.107
6	0.6	0.000145	5.5	0.107
7	0.6	0.000145	4.5	0.107
8	0.6	0.000145	4.0	0.107
9	0.6	0.000145	5.0	0.117
10	0.6	0.000145	5.0	0.127

5.3.1 Effect of sediment transport parameter α_1

Refering to equation 3.30 , the final equilibrium bed levels in the expansion do not depend upon a_1. The same can be observed from figure 5.15 . Here , three different values of a_1 equal to 0.000145, 0.000290 and 0.000435 are used. The equilibrium bed profile is same for all the three cases.





—— a_1 = 0.000145 + a_1 = 0.000290 a_1 = 0.000435

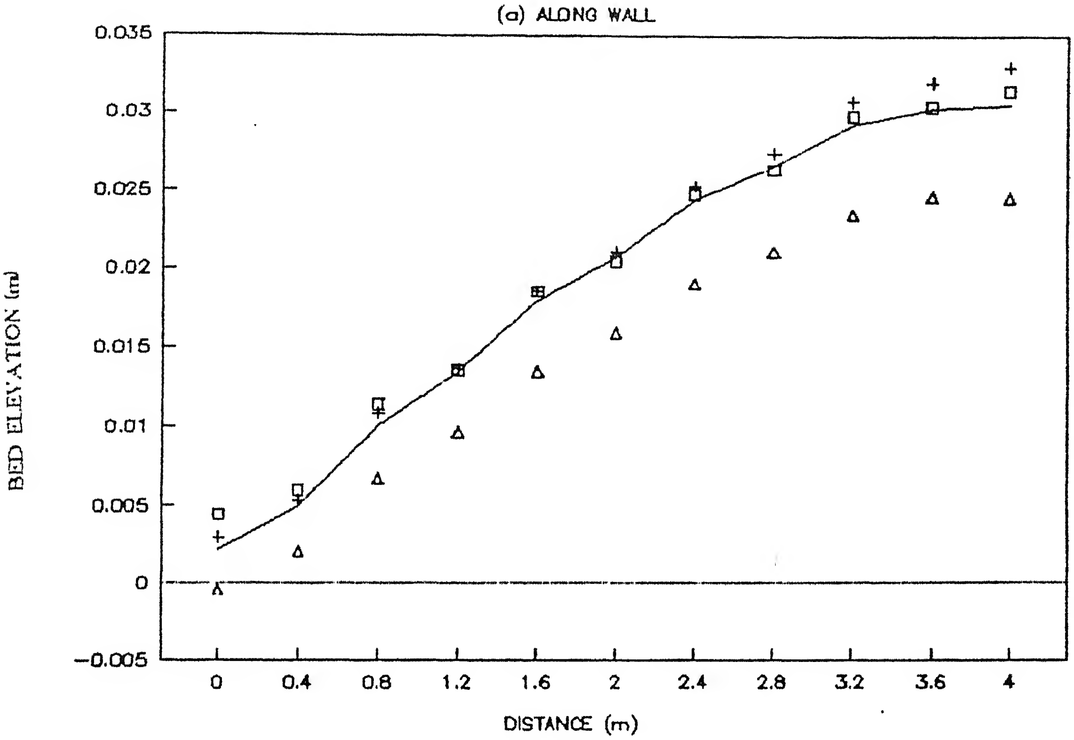
Figure 5.15 Effect of a_1 on bed elevation

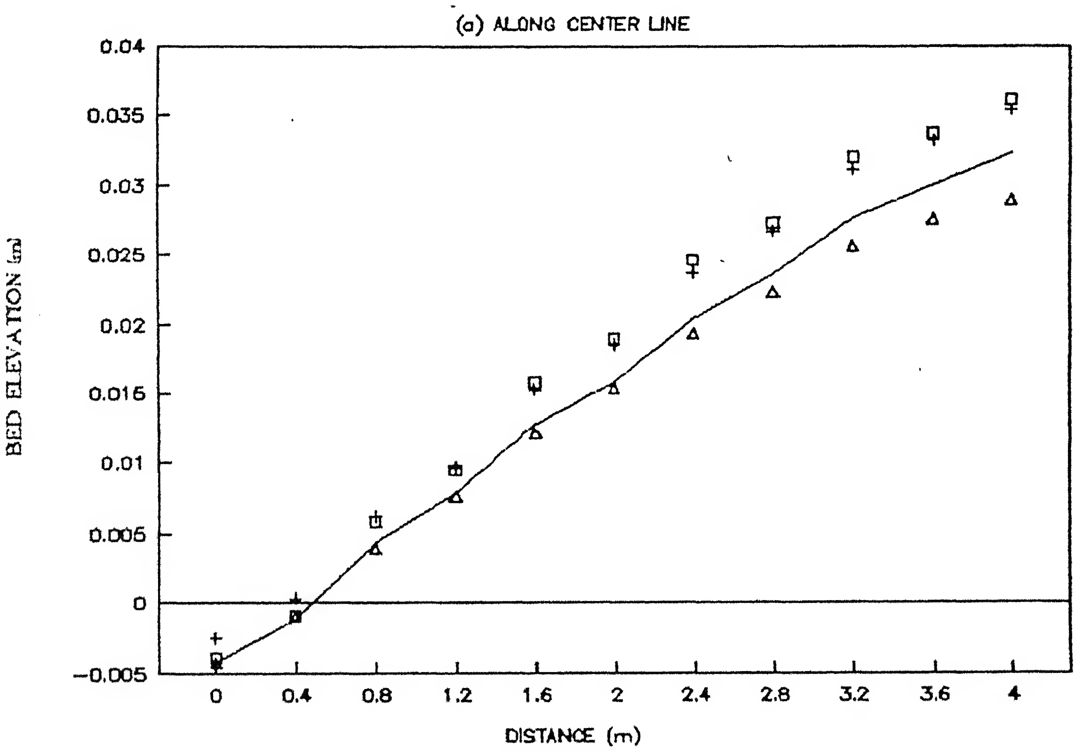
5.3.2 Effect of Sediment Transport Parameter a_2

Figure 5.16 shows the effect of a_2 on final equilibrium bed levels along the wall (figure 5.16 a) as well as along the center line (figure 5.16 b). Four different values of a_2 equal to 5.5, 5.0, 4.5 and 4.0 are considered. It is clearly indicated that larger the value of a_2 smaller the value of deposition or aggradation. If the flow parameters are such that the velocity is less than one, then a higher value of a_2 indicates less movement of sediment. Consequently, the aggradation will be less if all the other conditions are same. In the present case, the flow velocity is of the order of 0.2 m/s - 0.1 m/s . This explains the trends observed in figure 5.16 . However, the effect is highly non-linear as expected.

5.3.3 Effect of downstream flow depth, h_d

Figure 5.17 presents the variations of final equilibrium bed levels along the wall and along the centerline for three different cases of h_d equal to 0.107 m ,0.117 m and 0.127 m . For the computations made here, the downstream depth does not seem to have significant effect on the equilibrium bed profile. However, a higher downstream depth seems to increase the deposition (figure 5.17 b). A higher depth means a lower velocity and conseqently increased deposition.





a_2 = 4.0 + a_2 = 4.5 —— a_2 = 5.0 a_2 = 5.5

Figure 5.16 Effect of a_2 on bed elevation

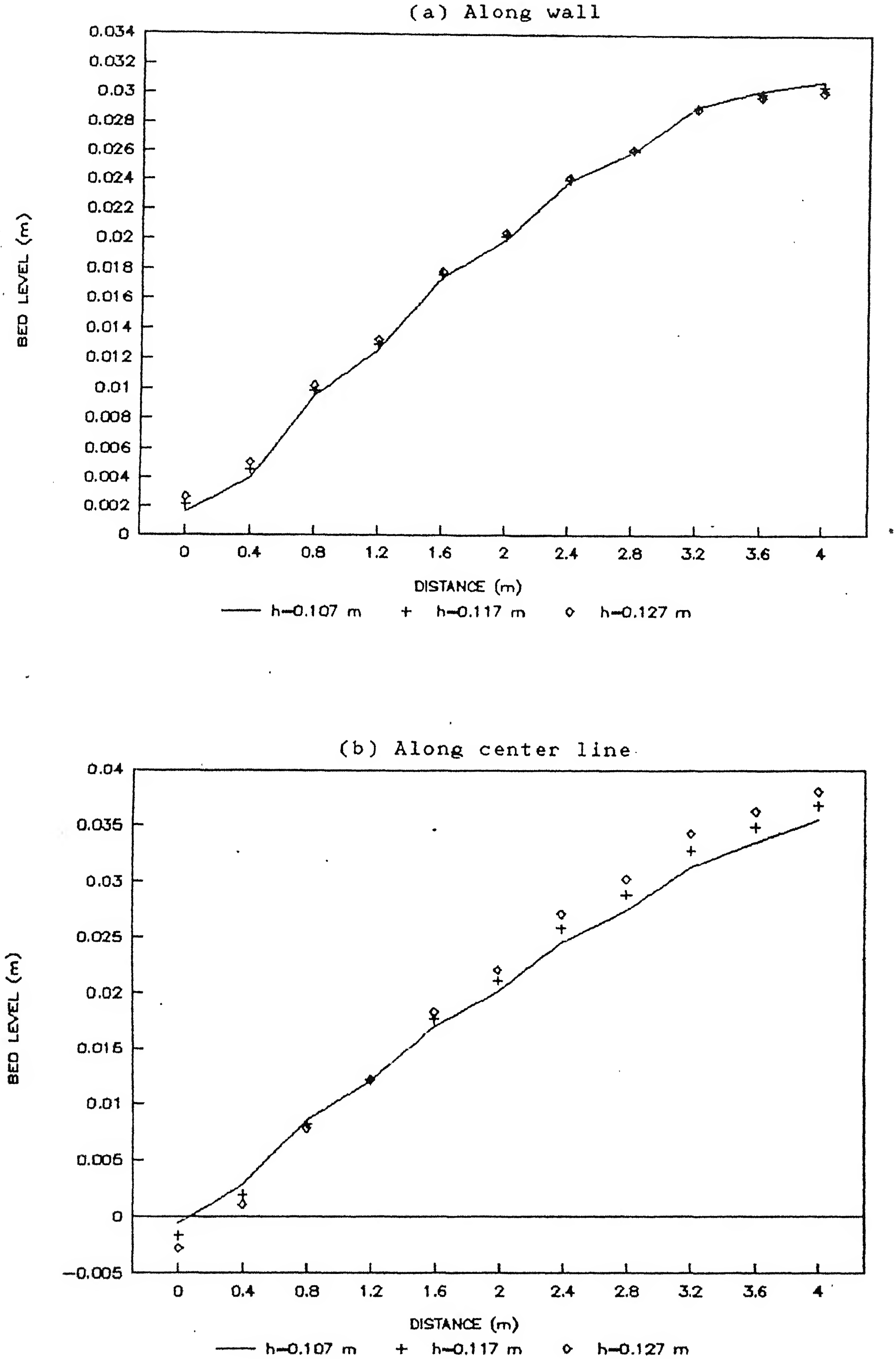


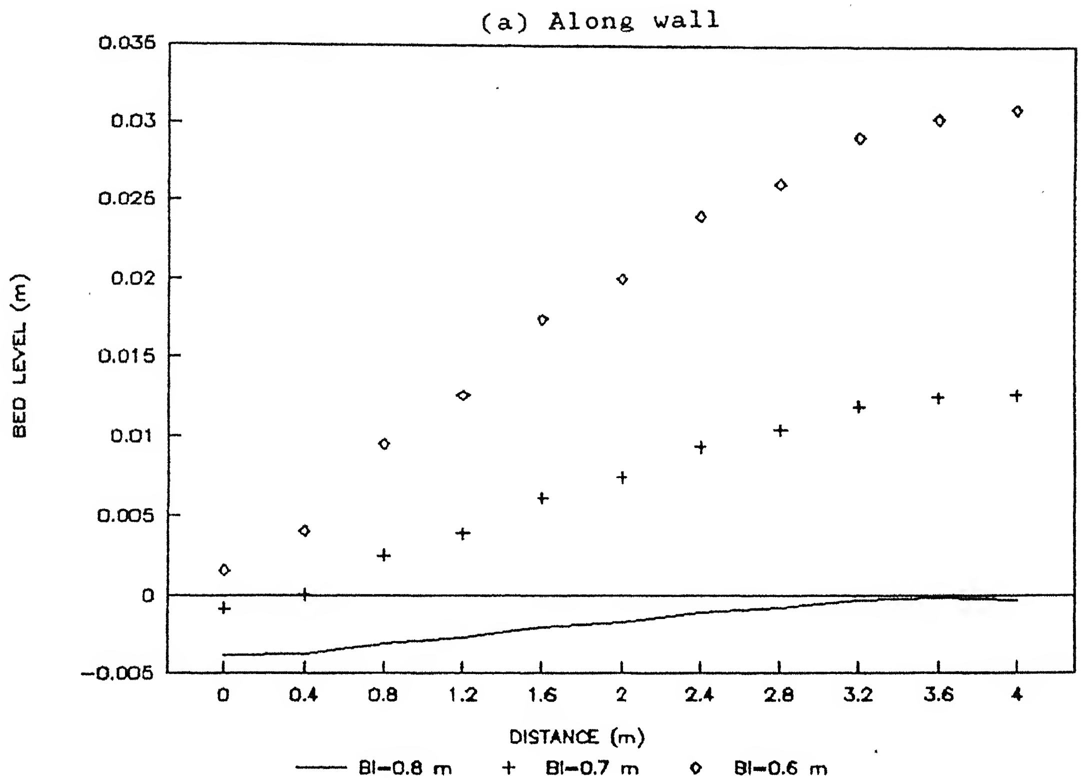
Figure 5.17 Effect of h_d on bed elevation

5.3.4 Effect of expansion ratio

Three different inlet widths, BI equal to 0.8 m, 0.7 m and 0.6 m are considered. Figure 5.18 shows the effect of BI or expansion ratio on the aggradation. As expected, higher expansion angle or a lower BI value results in more deposition and consequently higher equilibrium bed levels. As can be observed from figure 5.18, the deposition increases in the downstream direction.

5.4 TEMPORAL VARIATION OF BED LEVELS

The models presented in this study are verified by comparing numerical results with analytical results for final equilibrium bed levels. However, the main advantage of the numerical models is their ability to predict the temporal variation of bed levels. It is expected that the models give proper results even during the unsteady state. Unfortunately, it is not possible to verify this due to lack of analytical results. For the sake of completeness the temporal variation of channel bed profile is presented in figure 5.19 . Bed levels at time , t equal to 0.0 seconds (initial condition), 5 days, 25 days , 50 days and 100 days are shown. The final equilibrium bed profile is obtained at t = 100 days.



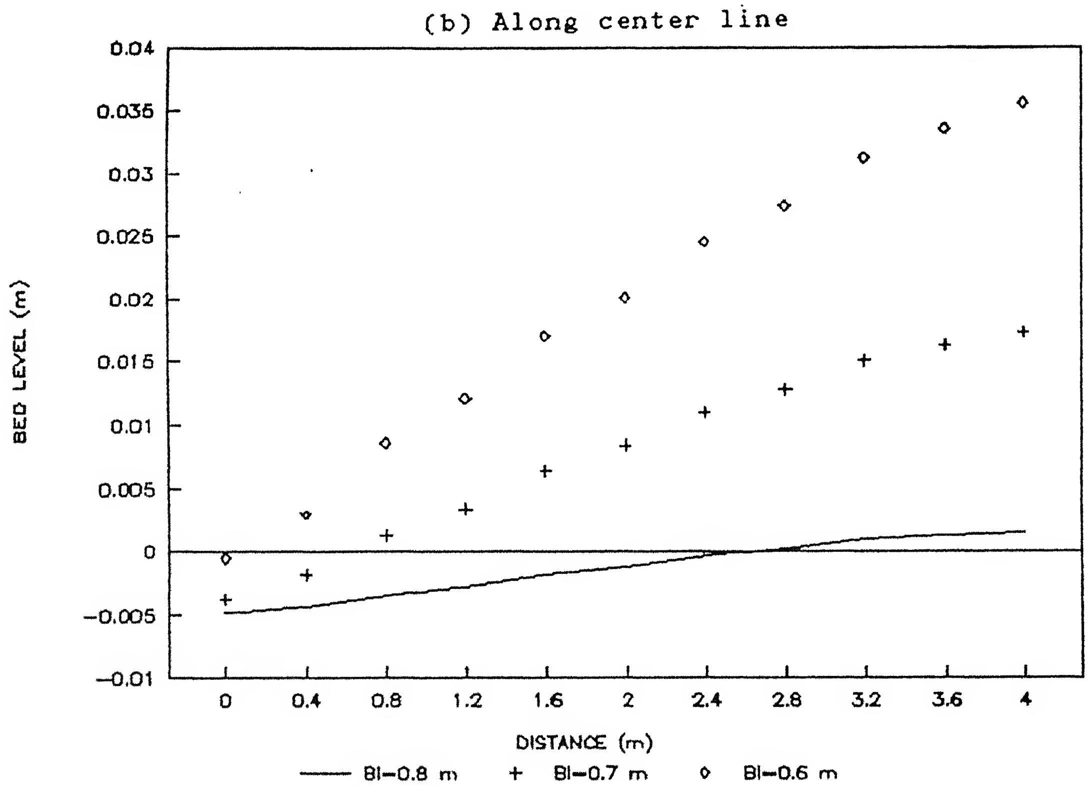


Figure 5.18 Effect of expansion ratio on bed elevation

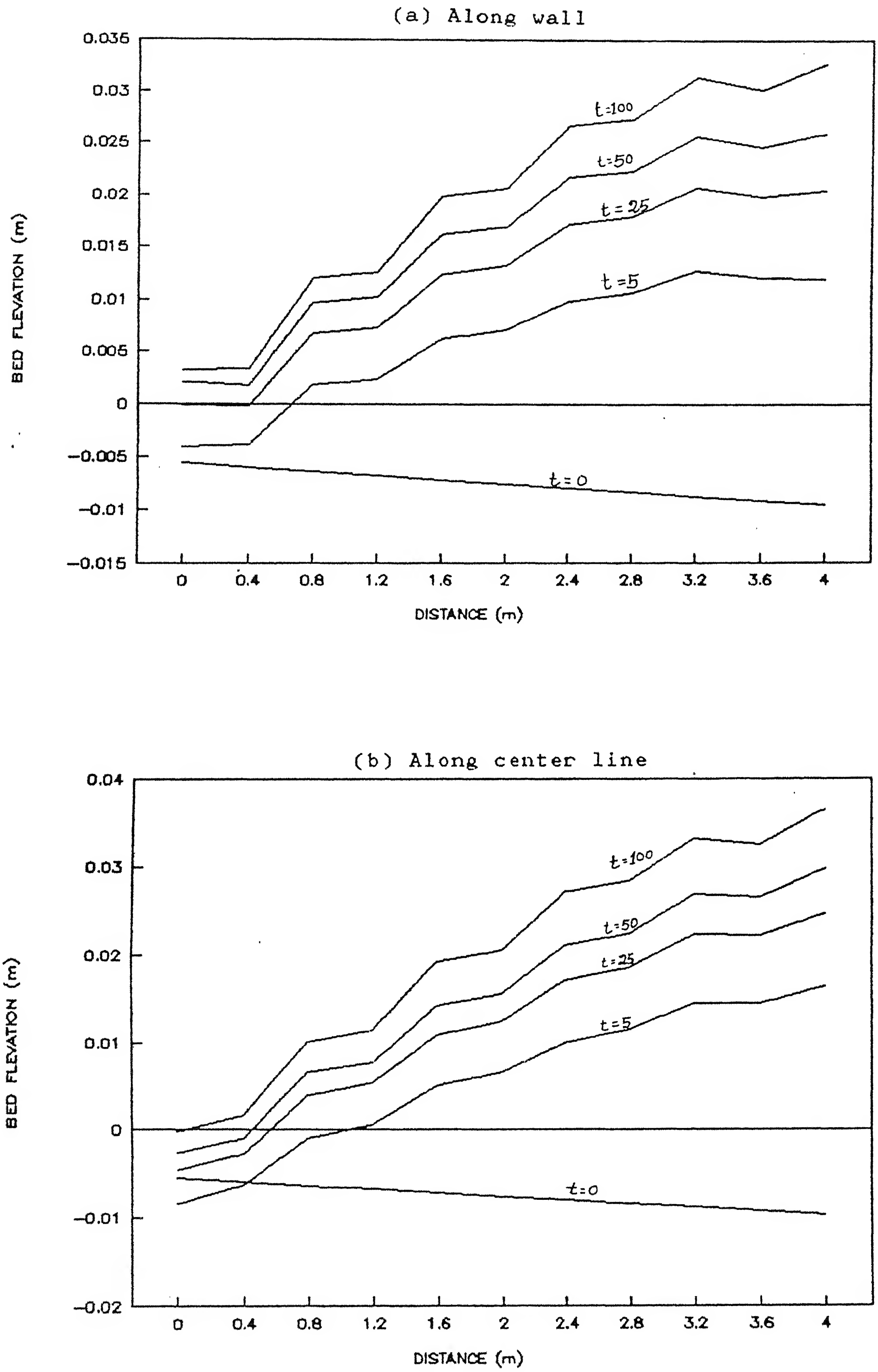


Figure 5.19 Temporal change in bed elevation

CHAPTER VI

Summary and Recommendations for Further Investigation

6.1 SUMMARY

This investigation presented analytical and numerical models for the analysis of flow and bed level variation in gradual open channel expansions with movable beds. Analytical solutions were developed for the equilibrium bed profile for steady conditions and no further aggradation. On the other hand, numerical models can predict both temporal and spatial variation of bed levels. A quqsi-steady , uncoupled approach was adopted in developing the one-dimensional as well as two-dimensional numerical models. The one-dimensional numerical results matched very well with the analytical results which are also for one-dimensional case. Two different numerical schemes were used in developing the two-dimensional models. Results from these two models matched with each other very well. The results were also comparable to those obtained using one-dimensional analytical

model. This indicated the satisfactory performance of the models as far as the solution of governing equations was conducted. However, the ~~s~~simple sediment transport equation used in the models forms a weak link. The sediment transport parameters should be properly evaluated before the models are applied in field situations. The models presented in this study were structured in such a way that any empirical sediment transport equation can be easily incorporated. A parametric study using the numerical models indicated the following.

1.) The sediment transport parameter a_1 have no erffect on the equilibrium bed profile.

2) An increase in the parameter a_2 decreases the equilibrium bed levels if the flow velocity is less than one.

3) The downstream flow depth slightly increases the deposition.

4) As expected, larger the expansion angle, greater is the deposition.

6.2 RECOMMENDATIONS FOR FURTHER WORK

The following is a list of subjects that require further research.

1) A completely unsteady coupled model needs to be developed for analysing the aggradation and degradation of bed levels in transitions during flood flows.

2) Several different boundary conditions should be included in the model for analysis of flow and bed in any general transition and for any upstream conditions like sediment over-loading.

3) A proper method for including the effect of armouring due to non-uniform size sediment needs to be explored.

4) There is no experimental data regarding the aggradation in channel expansions and there is an immediate need for such work.

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